

Capitolul 9

Structuri neliniare de calcul analogic

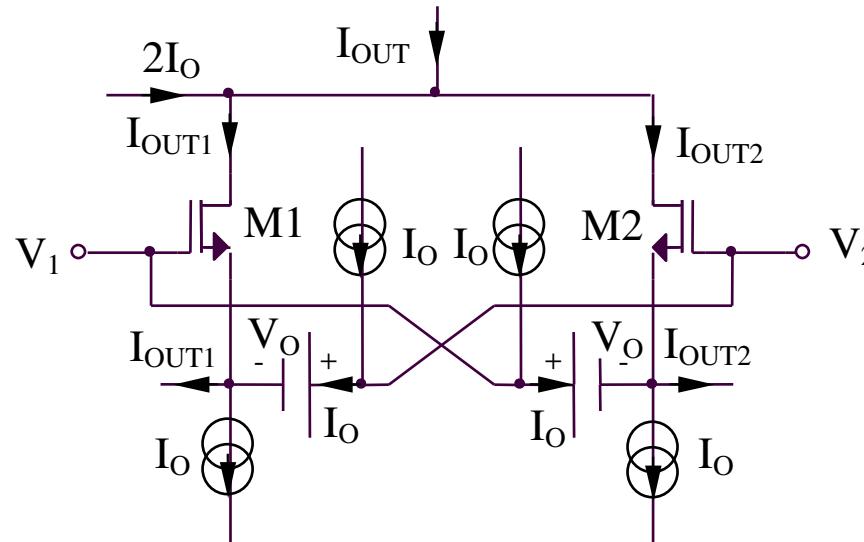
9.1. Structuri de ridicare la patrat

Circuite cu intrare in tensiune

9.1. Structuri de ridicare la patrat

9.1.1. Circuit de ridicare la patrat cu intrare in tensiune (I)

Schema de principiu



$$I_{OUT1} + I_{OUT2} = K(V_O - V_T)^2 + K(V_1 - V_2)^2$$

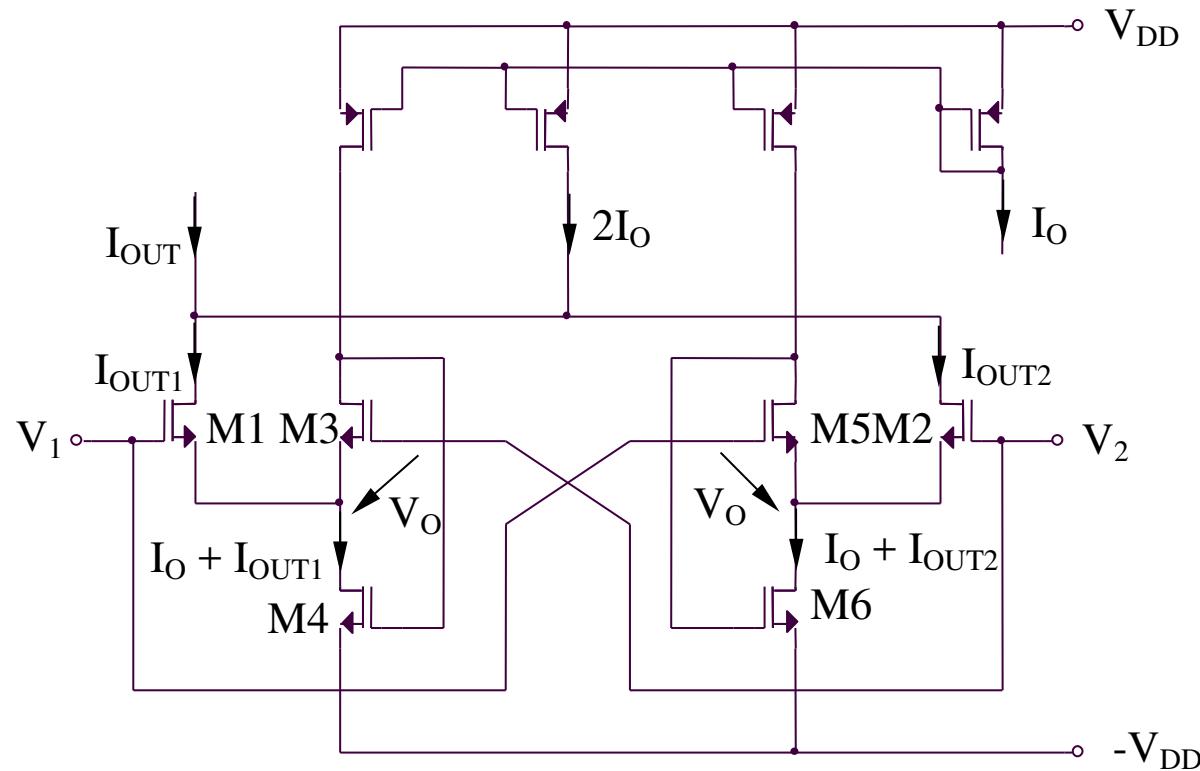
$$V_O = V_T + \sqrt{\frac{2I_O}{K}} \Rightarrow I_{OUT1} + I_{OUT2} = 2I_O + K(V_1 - V_2)^2$$

$$I_{OUT} = I_{OUT1} + I_{OUT2} - 2I_O = K(V_1 - V_2)^2$$

9.1. Structuri de ridicare la patrat

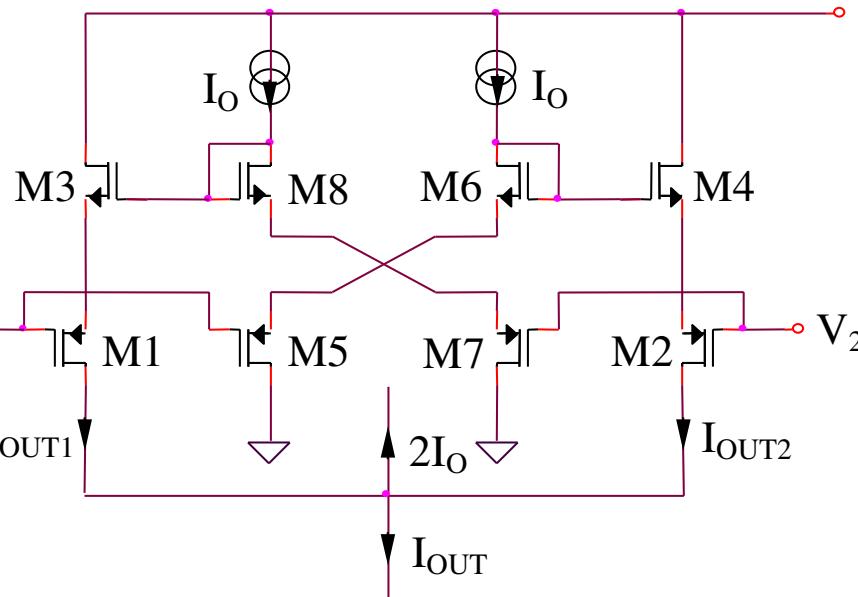
9.1.1. Circuit de ridicare la patrat cu intrare in tensiune (I) - continuare

Structura completa



9.1. Structuri de ridicare la patrat

9.1.2. Circuit de ridicare la patrat cu intrare in tensiune (II)



$$V_I - V_2 = 2V_{GS}(I_O) - 2V_{GS}(I_{OUT1}) = \\ = 2\sqrt{\frac{2}{K}}(\sqrt{I_O} - \sqrt{I_{OUT1}})$$

$$\Rightarrow \sqrt{I_{OUT1}} = \sqrt{I_O} - \frac{V_I - V_2}{2} \sqrt{\frac{K}{2}}$$

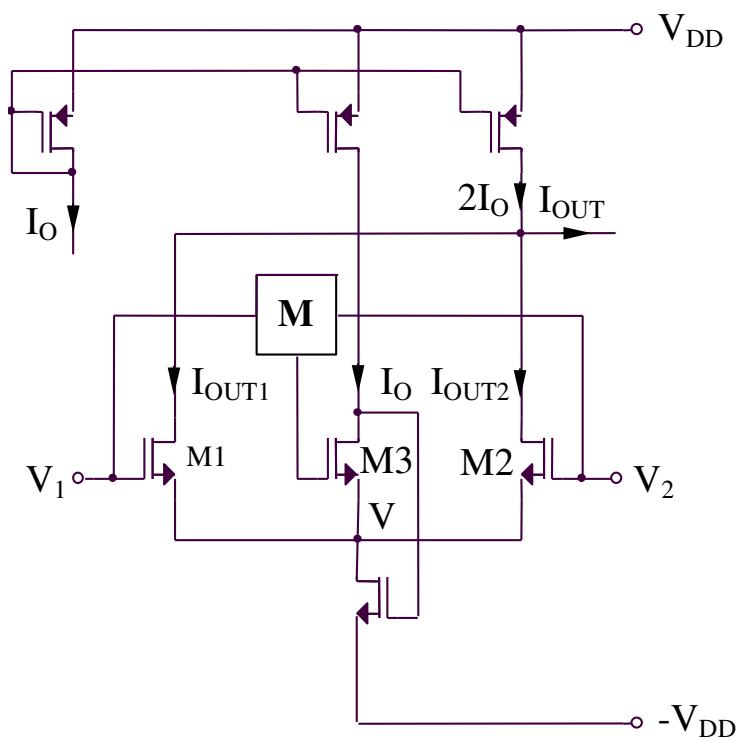
$$\Rightarrow I_{OUT1} = I_O - \sqrt{\frac{KI_O}{2}}(V_I - V_2) + \frac{K}{8}(V_I - V_2)^2$$

$$I_{OUT2} = I_O + \sqrt{\frac{KI_O}{2}}(V_I - V_2) + \frac{K}{8}(V_I - V_2)^2$$

$$I_{OUT} = I_{OUT1} + I_{OUT2} - 2I_O = \frac{K}{4}(V_I - V_2)^2$$

9.1. Structuri de ridicare la patrat

9.1.3. Circuit de ridicare la patrat cu intrare in tensiune (III)



$$I_{OUT1} = \frac{K}{2}(V_I - V - V_T)^2$$

$$I_{OUT2} = \frac{K}{2}(V_2 - V - V_T)^2$$

$$I_O = \frac{K}{2} \left(\frac{V_1 + V_2}{2} - V - V_T \right)^2 \Rightarrow$$

$$\Rightarrow V = \frac{V_1 + V_2}{2} - V_T - \sqrt{\frac{2I_O}{K}}$$

$$\Rightarrow I_{OUT1} = \frac{K}{2} \left(\frac{V_1 - V_2}{2} + \sqrt{\frac{2I_O}{K}} \right)^2$$

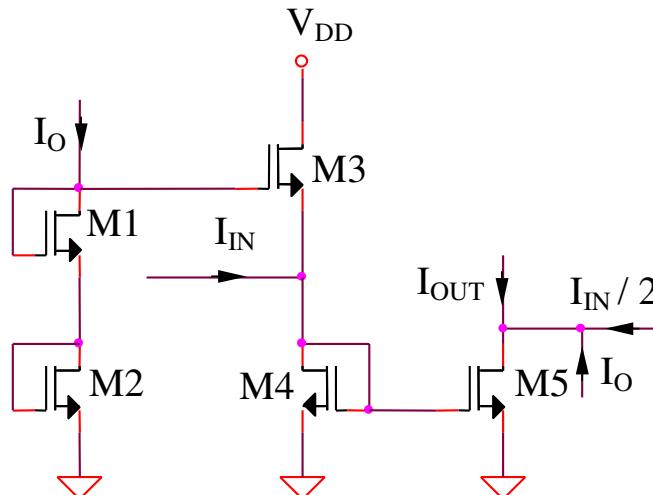
$$\Rightarrow I_{OUT2} = \frac{K}{2} \left(-\frac{V_1 - V_2}{2} + \sqrt{\frac{2I_O}{K}} \right)^2$$

$$I_{OUT} = I_{OUT1} + I_{OUT2} - 2I_O = \frac{K}{4}(V_1 - V_2)^2$$

Circuite cu intrare in curent

9.1. Structuri de ridicare la patrat

9.1.4. Circuit de ridicare la patrat cu intrare in curent (I)



$$V_{GS1} + V_{GS2} = V_{GS3} + V_{GS4}$$

$$2 \left(V_T + \sqrt{\frac{2I_O}{K}} \right) = V_T + \sqrt{\frac{2I_{D5}}{K}} + V_T + \sqrt{\frac{2(I_{D5} - I_{IN})}{K}}$$

$$2\sqrt{I_O} = \sqrt{I_{D5}} + \sqrt{I_{D5} - I_{IN}}$$

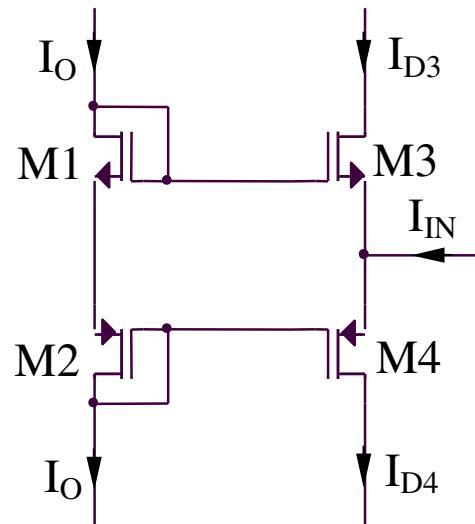
$$I_{D5} = I_O + \frac{I_{IN}}{2} + \frac{I_{IN}^2}{16I_O}$$

$$I_{OUT} = I_{D5} - I_O - \frac{I_{IN}}{2} = \frac{I_{IN}^2}{16I_O}$$

9.1. Structuri de ridicare la patrat

9.1.5. Circuit de ridicare la patrat cu intrare in curent (II)

Nucleul functional



$$V_{GS1} + V_{SG2} = V_{GS3} + V_{SG4}$$

$$2\sqrt{I_O} = \sqrt{I_{D3}} + \sqrt{I_{D3} + I_{IN}}$$

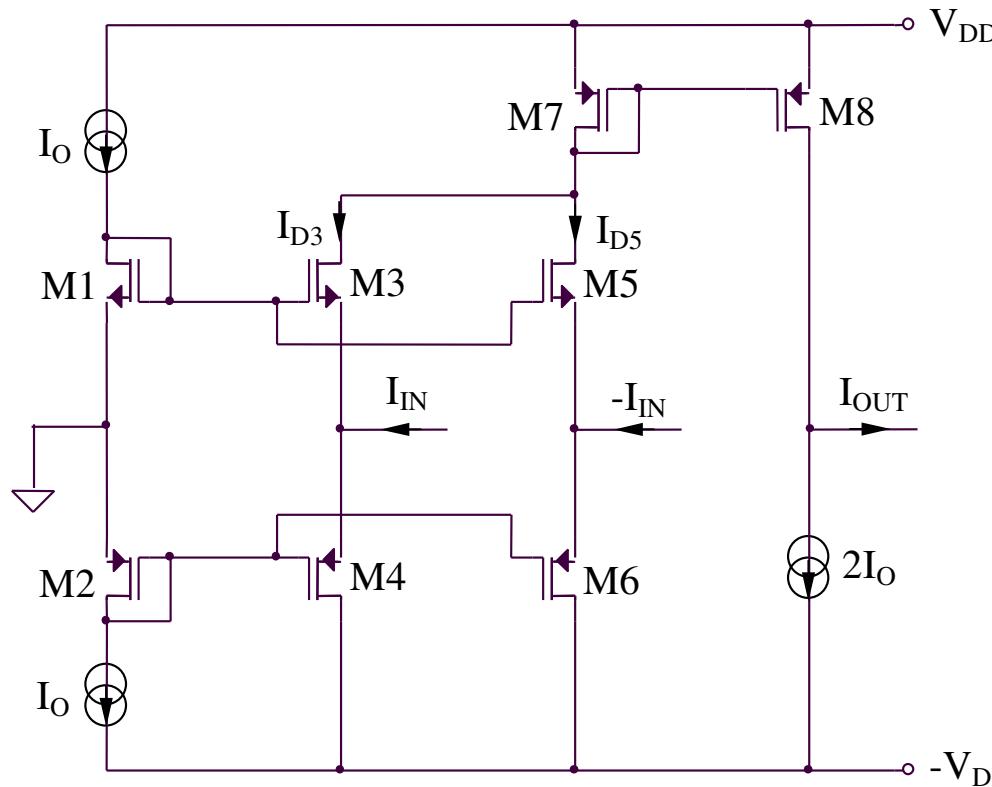
$$4I_O = 2I_{D3} + I_{IN} + 2\sqrt{I_{D3}(I_{D3} + I_{IN})}$$

$$I_{D3} = I_O - \frac{I_{IN}}{2} + \frac{I_{IN}^2}{16I_O}$$

9.1. Structuri de ridicare la patrat

9.1.5. Circuit de ridicare la patrat cu intrare in curent (II)

Circuitul complet



$$I_{D3} = I_O - \frac{I_{IN}}{2} + \frac{I_{IN}^2}{16I_O}$$

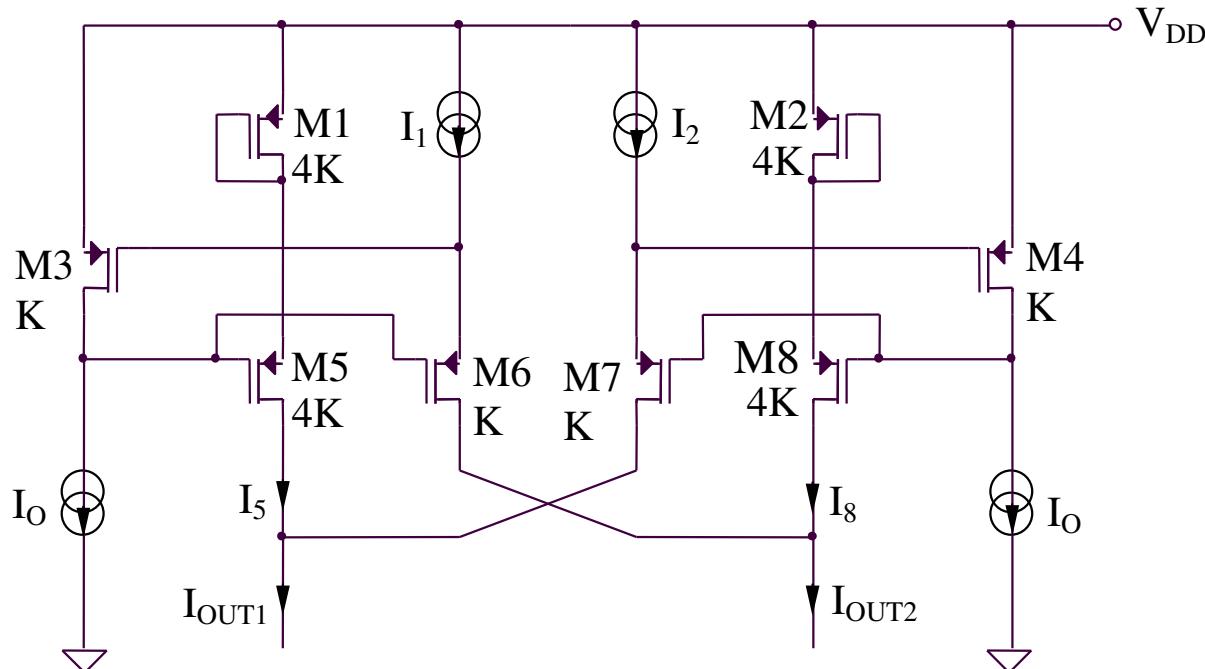
$$I_{D5} = I_O + \frac{I_{IN}}{2} + \frac{I_{IN}^2}{16I_O}$$

$$I_{OUT} = I_{D3} + I_{D5} - 2I_O = \frac{I_{IN}^2}{8I_O}$$

9.2. Structuri de extragere a radacinii patrate

9.2. Structuri de extragere a radacinii patrate

9.2.1 Circuit de extragere a radacinii patrate (I)



$$V_{SG3} + V_{SG6} = V_{SG1} + V_{SG5}$$

echivalent cu:

$$2\sqrt{\frac{2I_5}{4K}} = \sqrt{\frac{2I_O}{K}} + \sqrt{\frac{2I_1}{K}}$$

rezulta:

$$I_5 = I_O + I_1 + 2\sqrt{I_O I_1}$$

Similar:

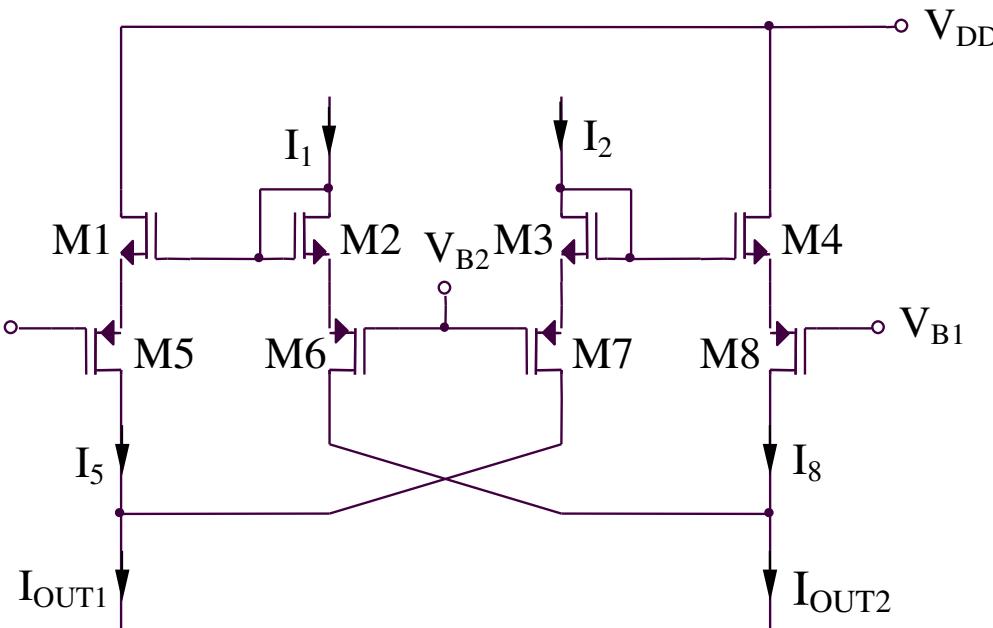
$$I_8 = I_O + I_2 + 2\sqrt{I_O I_2}$$

Deci:

$$I_{OUT1} - I_{OUT2} = (I_5 + I_2) - (I_8 + I_1) = 2\sqrt{I_O} (\sqrt{I_1} - \sqrt{I_2})$$

9.2. Structuri de extragere a radacinii patrate

9.2.2 Circuit de extragere a radacinii patrate (II)



$$V_{B2} - V_{B1} = 2V_{GS}(I_5) - 2V_{GS}(I_1)$$

echivalent cu:

$$V_{B2} - V_{B1} = 2\sqrt{\frac{2}{K}}(\sqrt{I_5} - \sqrt{I_1})$$

rezulta:

$$\sqrt{I_5} = \sqrt{I_1} + \sqrt{\frac{K}{8}}(V_{B2} - V_{B1})$$

Deci:

$$I_5 = I_1 + \sqrt{\frac{KI_1}{2}}(V_{B2} - V_{B1}) + \frac{K}{8}(V_{B2} - V_{B1})^2$$

Similar:

$$I_8 = I_2 + \sqrt{\frac{KI_2}{2}}(V_{B2} - V_{B1}) + \frac{K}{8}(V_{B2} - V_{B1})^2$$

In concluzie:

$$I_{OUT1} - I_{OUT2} = (I_5 + I_2) - (I_8 + I_1) = \sqrt{\frac{K}{2}}(V_{B2} - V_{B1})(\sqrt{I_1} - \sqrt{I_2})$$

9.3. Structuri pentru realizarea functiei exponentiale

9.3. Structuri pentru realizarea functiei exponentiale

Pentru obtinerea unui raspuns in frecventa superior, se impune utilizarea exclusiva a tranzistoarelor MOS polarizate in saturatie.

Exista posibilitatea de a aproxima functia exponentiala utilizandu-se:

a. Dezvoltarea sa in serie Taylor limitata $g(x)$:

$$g(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \cong \exp(x)$$

b. Functii de aproximare de ordin superior. Exemple:

- de ordin II:

$$g_{II}(x) = \frac{1 + \frac{x}{2}}{1 - \frac{x}{2}} \cong \exp(x)$$

- de ordin III:

$$g_{III}(x) = \frac{1 + \frac{7x}{6}}{1 - \frac{x}{3}} - \frac{x}{2} \cong \exp(x)$$

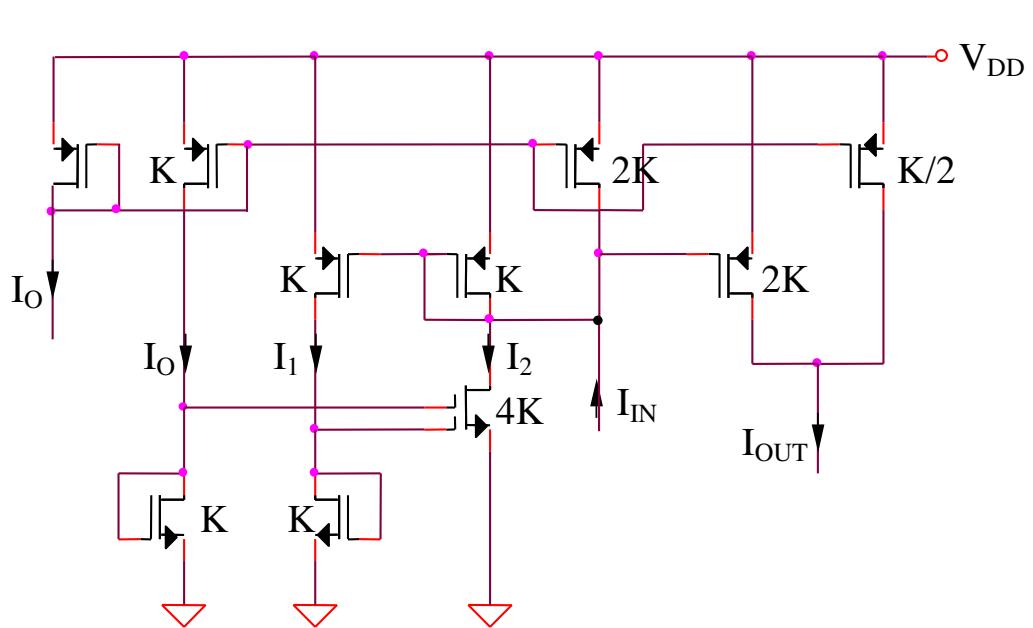
- de ordin IV:

$$g_{IV}(x) = \frac{1 + x + \frac{x^2}{3}}{1 - x + \frac{x^2}{3}} \cong \exp(x)$$

9.3. Structuri pentru realizarea functiei exponentiale

9.3.1. Circuit de exponentiere utilizand dezvoltare in serie Taylor de ordin II

$$I_O \exp\left(\frac{I_{IN}}{I_O}\right) \cong I_O \left[1 + \frac{I_{IN}}{I_O} + \frac{1}{2} \left(\frac{I_{IN}}{I_O} \right)^2 + \dots \right]$$



$$I_2 = \frac{4K}{2} \left[\frac{1}{2} \left(2V_T + \sqrt{\frac{2I_1}{K}} + \sqrt{\frac{2I_O}{K}} \right) - V_T \right]^2$$

echivalent cu:

$$I_2 = I_1 + I_O + 2\sqrt{I_1 I_O}$$

Deoarece:

$$I_2 = I_1 + 2I_O + I_{IN}$$

rezulta:

$$I_1 = \frac{(I_O + I_{IN})^2}{4I_O} = \frac{I_O}{4} + \frac{I_{IN}}{2} + \frac{I_{IN}^2}{4I_O}$$

Curentul de iesire va avea expresia urmatoare, deci va aproxima functia exponentiala:

$$I_{OUT} = 2I_1 + \frac{I_O}{2} = I_O + I_{IN} + \frac{I_{IN}^2}{2I_O}$$

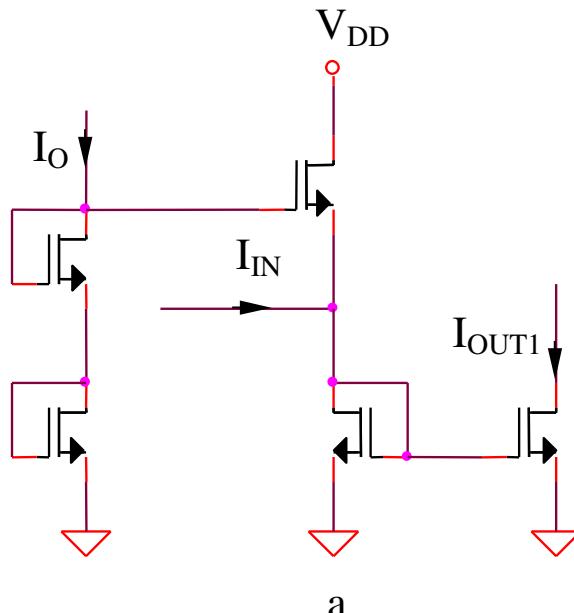
$$I_{OUT} \cong I_O \exp\left(\frac{I_{IN}}{I_O}\right)$$

9.3. Structuri pentru realizarea functiei exponentiale

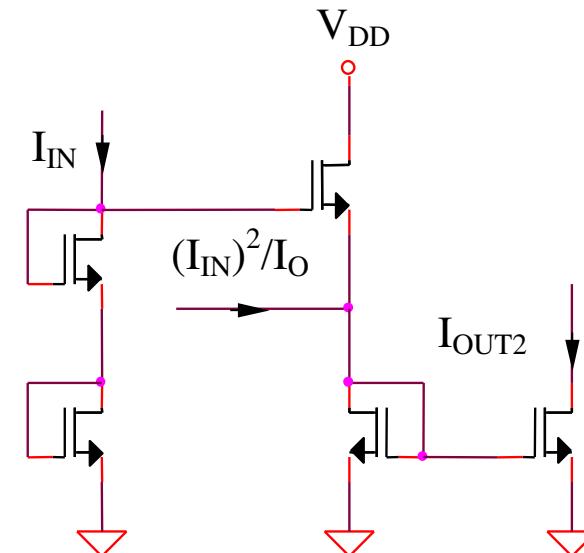
9.3.2. Circuit de exponentiere utilizand dezvoltare in serie Taylor de ordin III

$$I_O \exp\left(\frac{I_{IN}}{I_O}\right) \approx I_O \left[1 + \frac{I_{IN}}{I_O} + \frac{1}{2} \left(\frac{I_{IN}}{I_O}\right)^2 + \frac{1}{6} \left(\frac{I_{IN}}{I_O}\right)^3 + \dots \right]$$

Nucleu functional



a



b

$$2V_{GS}(I_O) = V_{GS}(I_{OUT1}) + V_{GS}(I_{OUT1} - I_{IN})$$

implica:

$$2\sqrt{I_O} = \sqrt{I_{OUT1}} + \sqrt{I_{OUT1} - I_{IN}}$$

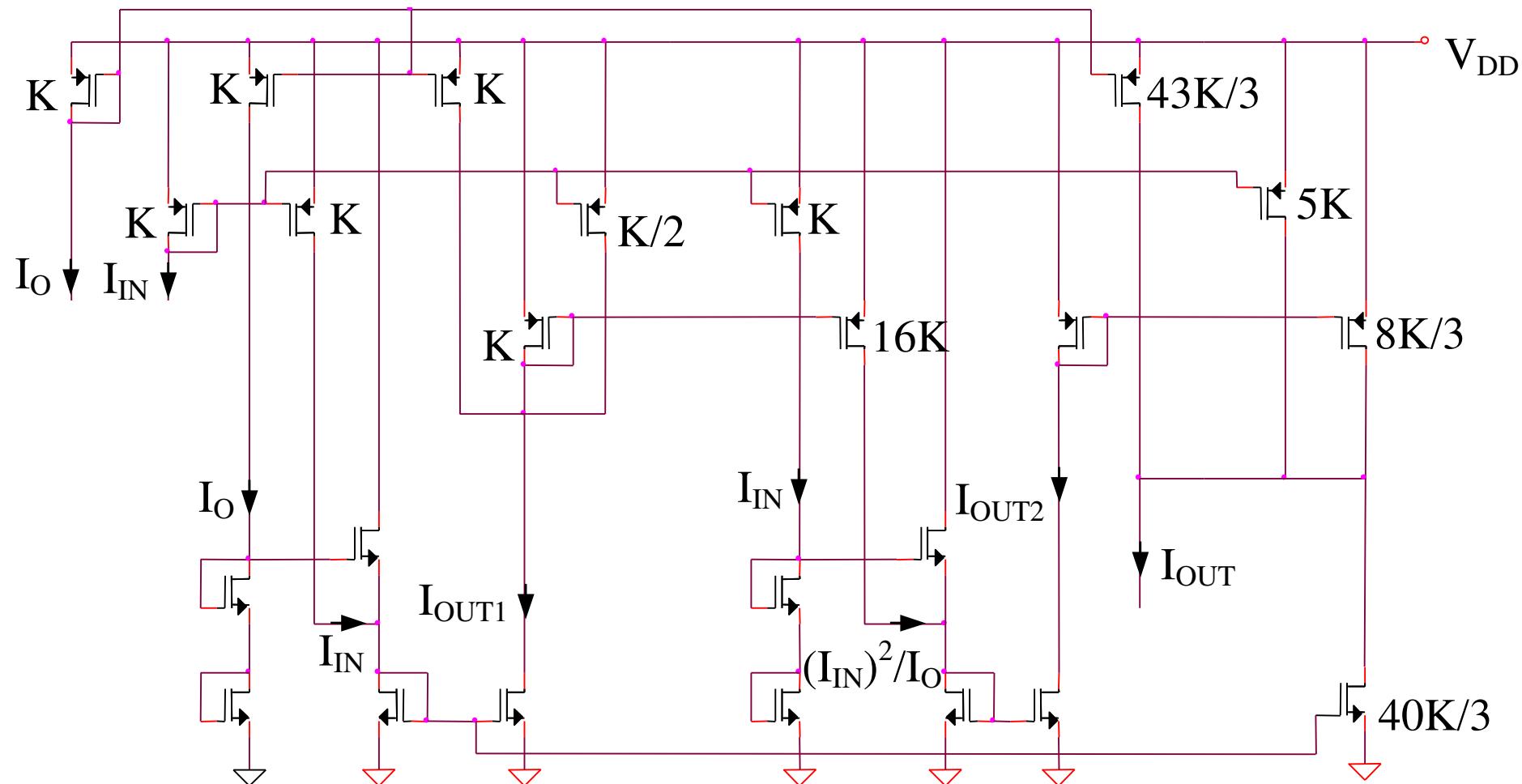
$$\Rightarrow \begin{cases} I_{OUT1} = I_O + \frac{I_{IN}}{2} + \frac{I_{IN}^2}{16I_O} \\ I_{OUT2} = I_{IN} + \frac{I_{IN}^2}{2I_O} + \frac{I_{IN}^3}{16I_O^2} \end{cases}$$

9.3. Structuri pentru realizarea functiei exponentiale

9.3.2. Circuit de exponentiere utilizand dezvoltare in serie Taylor de ordin III

(continuare)

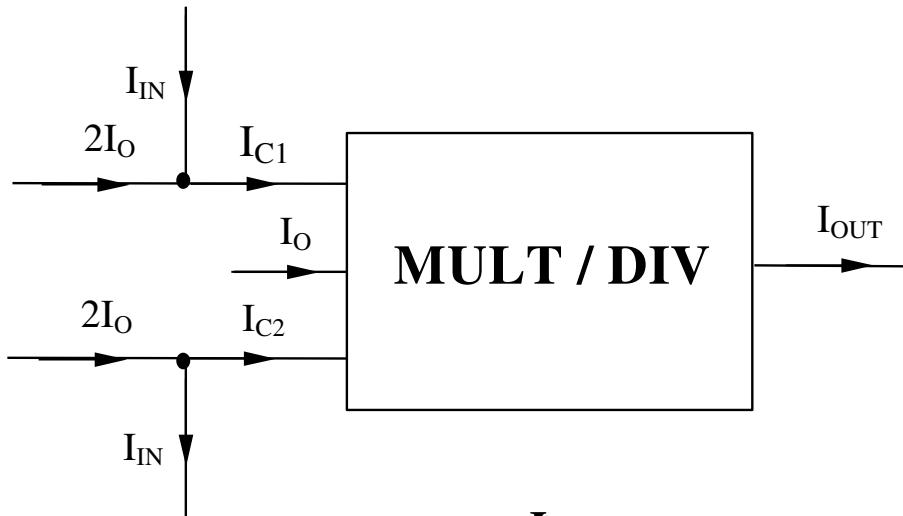
Circuit complet



$$I_{OUT} = I_O + \frac{40}{3}(I_O - I_{OUT1}) + 5I_{IN} + \frac{8}{3}I_{OUT2} \approx I_O \exp\left(\frac{I_{IN}}{I_O}\right)$$

9.3. Structuri pentru realizarea functiei exponentiale

9.3.3. Circuit de exponentiere utilizand functii de aproximare de ordin II



$$I_{OUT} = I_O \frac{I_{C1}}{I_{C2}}$$

Rezulta:

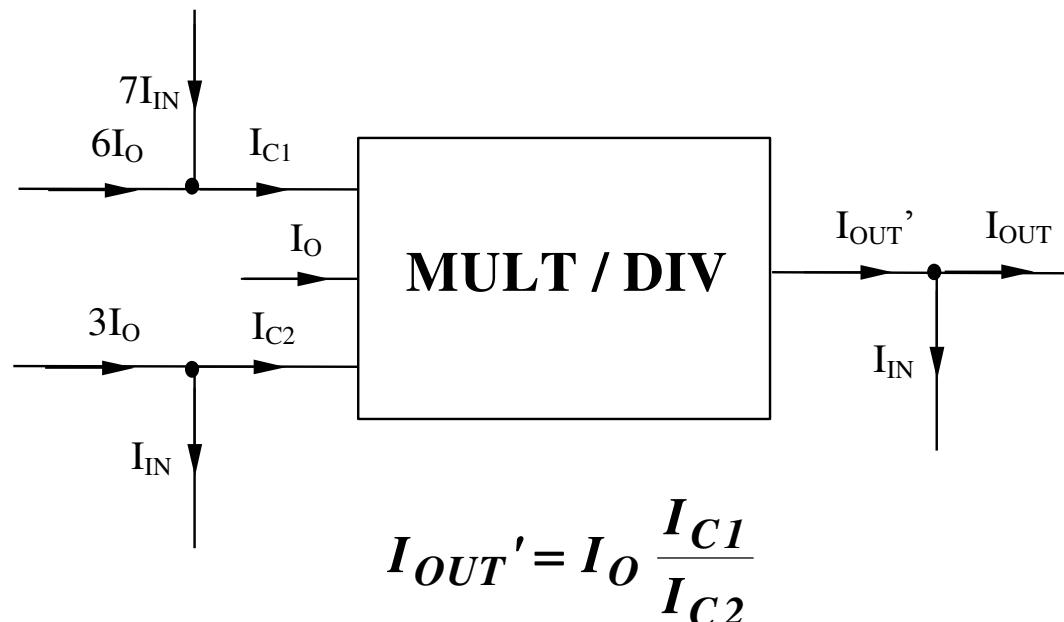
$$I_{OUT} = I_O \frac{2I_O + I_{IN}}{2I_O - I_{IN}} = I_O \frac{1 + \frac{1}{2} \left(\frac{I_{IN}}{I_O} \right)}{1 - \frac{1}{2} \left(\frac{I_{IN}}{I_O} \right)} = I_O g_{II} \left(\frac{I_{IN}}{I_O} \right)$$

Deci, I_{OUT} va fi proportional (intr-o aproximare de ordin II) cu functia exponentiala:

$$I_{OUT} \approx I_O \exp \left(\frac{I_{IN}}{I_O} \right)$$

9.3. Structuri pentru realizarea functiei exponentiale

9.3.4. Circuit de exponentiere utilizand functii de aproximare de ordin III



Rezulta:

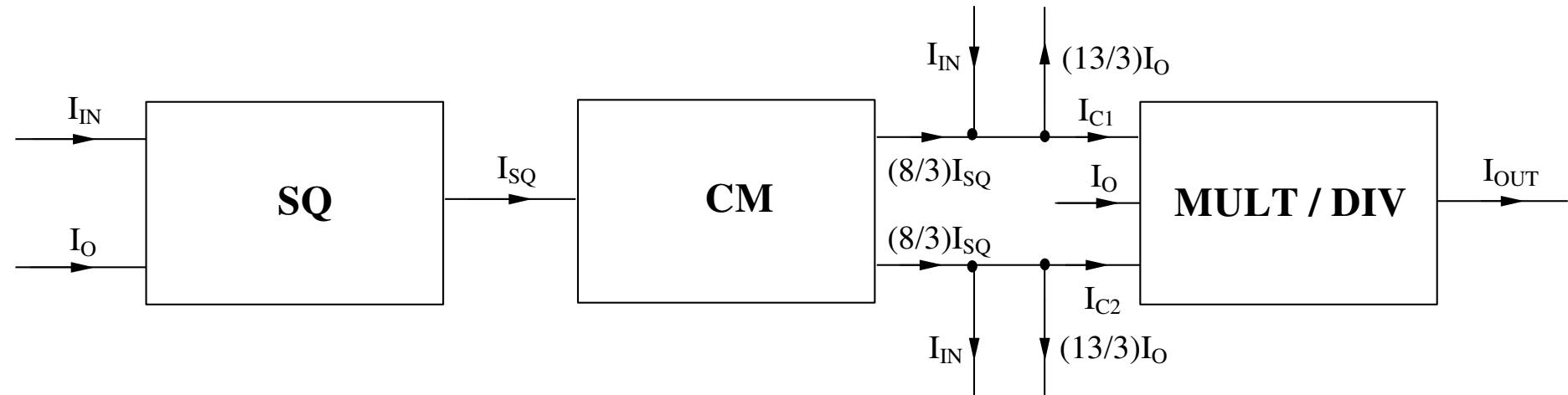
$$I_{OUT} = I_{OUT'} - I_{IN} = 2I_O \left[\frac{1 + \frac{7}{6} \left(\frac{I_{IN}}{I_O} \right)}{1 - \frac{1}{3} \left(\frac{I_{IN}}{I_O} \right)} - \frac{1}{2} \left(\frac{I_{IN}}{I_O} \right) \right] = 2I_O g_{III} \left(\frac{I_{IN}}{I_O} \right)$$

Deci, I_{OUT} va fi proportional (intr-o aproximare de ordin III) cu functia exponentiala:

$$I_{OUT} \cong 2I_O \exp \left(\frac{I_{IN}}{I_O} \right)$$

9.3. Structuri pentru realizarea functiei exponentiale

9.3.5. Circuit de exponentiere utilizand functii de aproximare de ordin IV



$$I_{OUT} = I_O \frac{I_{C1}}{I_{C2}}$$

$$I_{SQ} = 2I_O + \frac{I_{IN}^2}{8I_O}$$

Rezulta:

$$I_{OUT} = I_O \frac{I_{C1}}{I_{C2}} = \frac{\frac{8}{3}I_{SQ} + I_{IN} - \frac{13}{3}I_O}{\frac{8}{3}I_{SQ} - I_{IN} - \frac{13}{3}I_O} = I_O \frac{I_O + I_{IN} + \frac{I_{IN}^2}{3I_O}}{I_O - I_{IN} + \frac{I_{IN}^2}{3I_O}} = I_O g_{IV} \left(\frac{I_{IN}}{I_O} \right)$$

Deci, I_{OUT} va fi proportional (intr-o aproximare de ordin IV) cu functia exponentiala:

$$I_{OUT} \approx I_O \exp \left(\frac{I_{IN}}{I_O} \right)$$

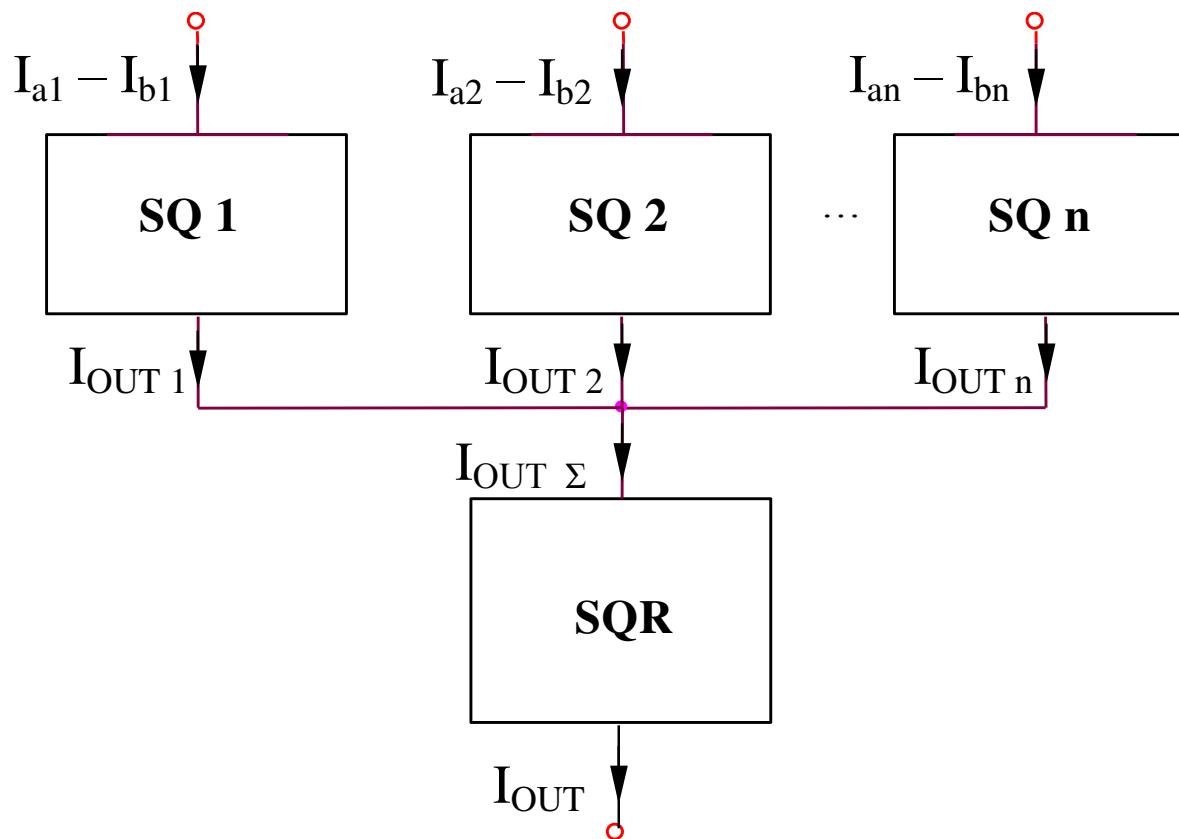
9.4. Structuri pentru realizarea functiei euclidiene

9.4. Structuri pentru realizarea functiei euclidiene

$$I_{OUT} = \sqrt{\frac{1}{n} \sum_{k=1}^n (I_{ak} - I_{bk})^2} = \sqrt{\frac{1}{n} \sum_{k=1}^n I_{INk}^2}$$

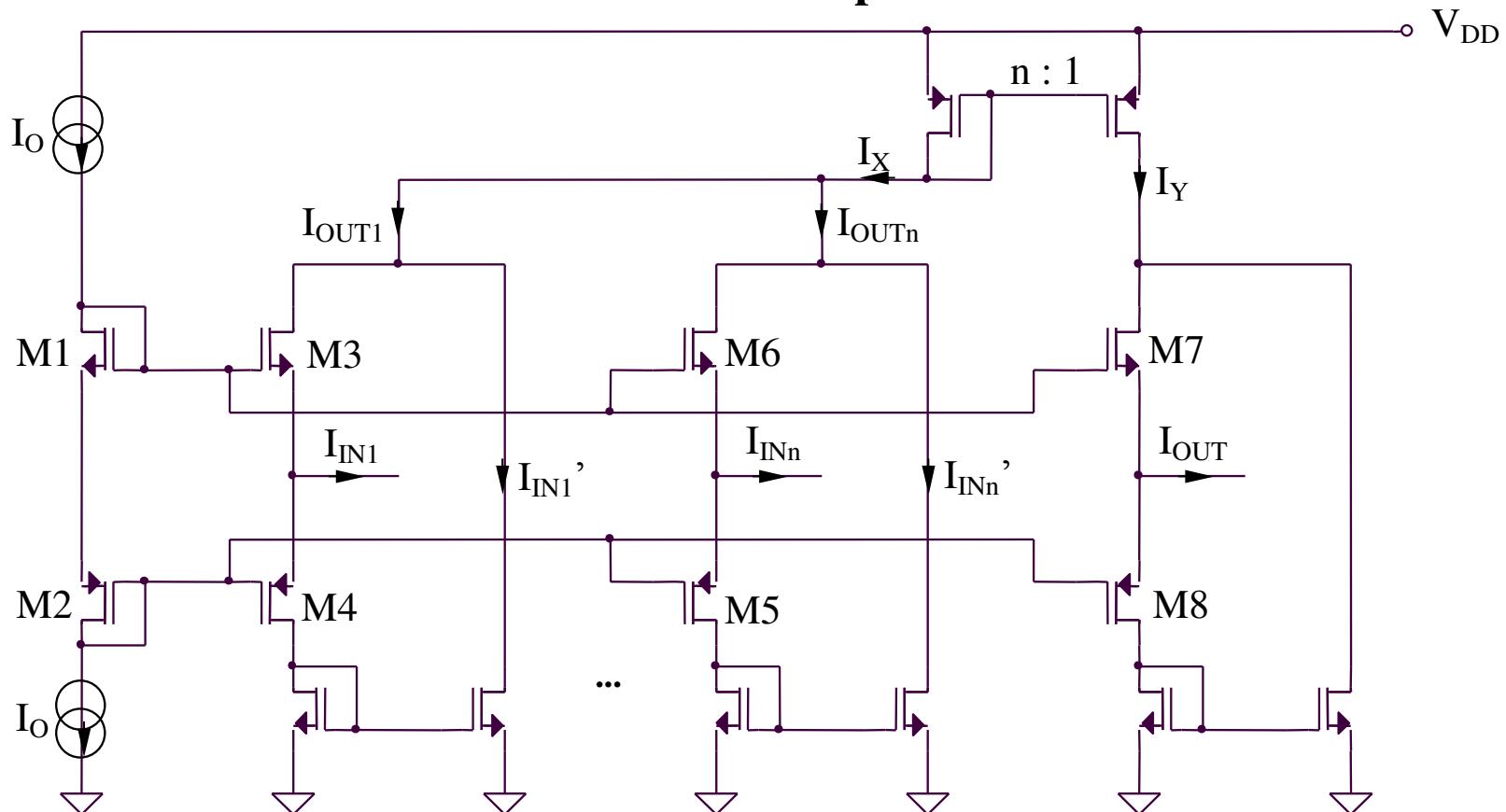
$$I_{INk} = I_{ak} - I_{bk}$$

Schema bloc



9.4. Structuri pentru realizarea functiei euclidiene

Circuit complet



$$V_{GS1} + V_{SG2} = V_{GS3} + V_{SG4} \quad \mid \Rightarrow \\ I_{IN1} + I_{IN1'} = I_{OUT1} - I_{IN1'} \Rightarrow I_{IN1'} = \frac{I_{OUT1} - I_{IN1}}{2}$$

$$\Rightarrow 2\sqrt{I_O} = \sqrt{\frac{I_{OUT1} + I_{IN1}}{2}} + \sqrt{\frac{I_{OUT1} - I_{IN1}}{2}} \Rightarrow I_{OUT1} = 2I_O + \frac{I_{IN1}^2}{8I_O}$$

9.4. Structuri pentru realizarea functiei euclidiene

Deci:

$$I_{OUT1} = 2I_O + \frac{I_{IN1}^2}{8I_O}$$

Similar:

$$I_{OUTn} = 2I_O + \frac{I_{INn}^2}{8I_O}$$

Dar:

$$I_X = \sum_{k=1}^n I_{OUTk} = 2nI_O + \frac{1}{8I_O} \sum_{k=1}^n I_{INk}^2$$

Pe de alta parte:

$$I_Y = \frac{I_X}{n} = 2I_O + \frac{1}{8nI_O} \sum_{k=1}^n I_{INk}^2$$

$$I_Y = 2I_O + \frac{I_{OUT}^2}{8I_O}$$

rezultand:

$$I_{OUT} = \sqrt{\frac{1}{n} \sum_{k=1}^n I_{INk}^2}$$