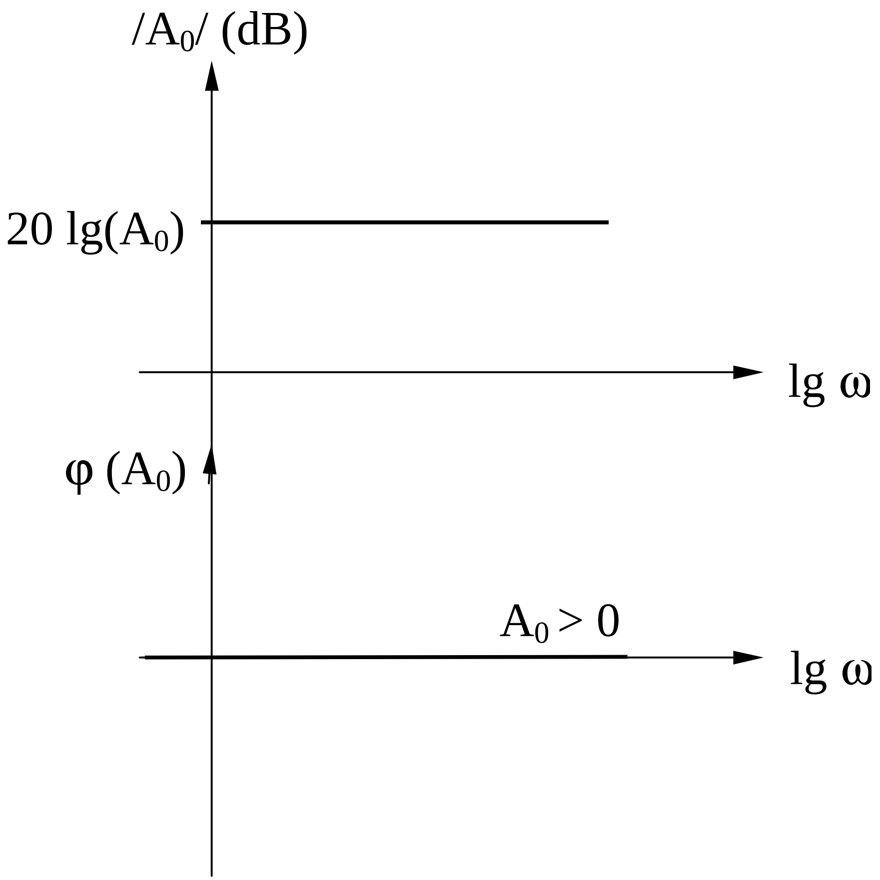
Capitolul 7
Raspunsul in frecventa al circuitelor. Stabilitatea circuitelor cu reactie

7.1. Caracteristicile de frecventa ale functiilor elementare. Diagrame Bode

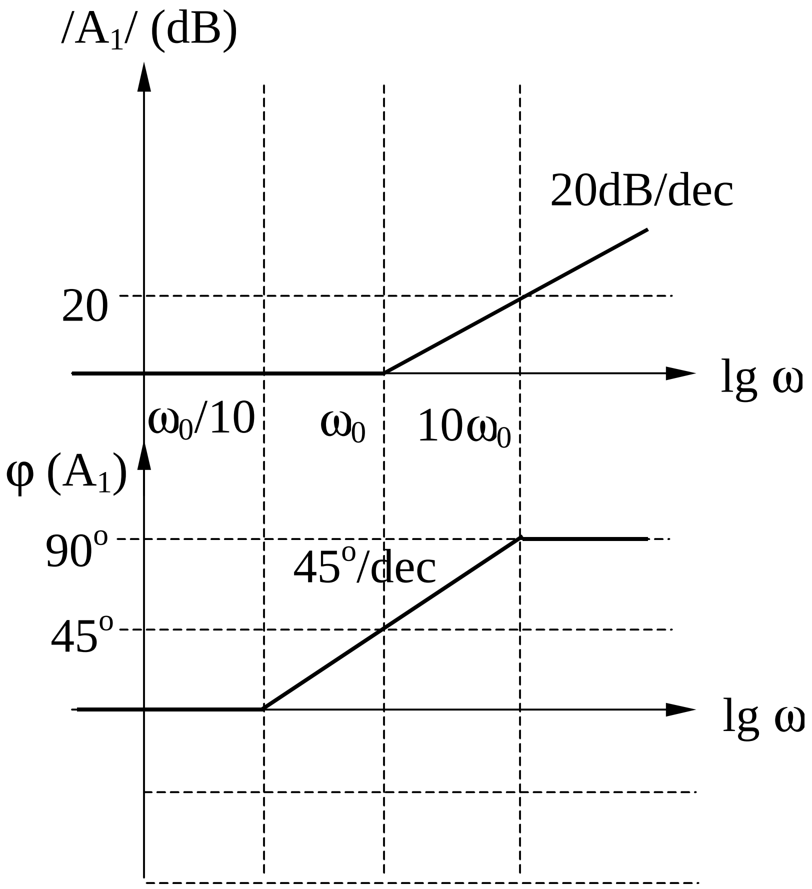
7.1. Caracteristicile de frecventa ale functiilor elementare. Diagrame Bode

Constanta (A_0)



$$A_0 = ct.$$

Zero real negativ (A_1)

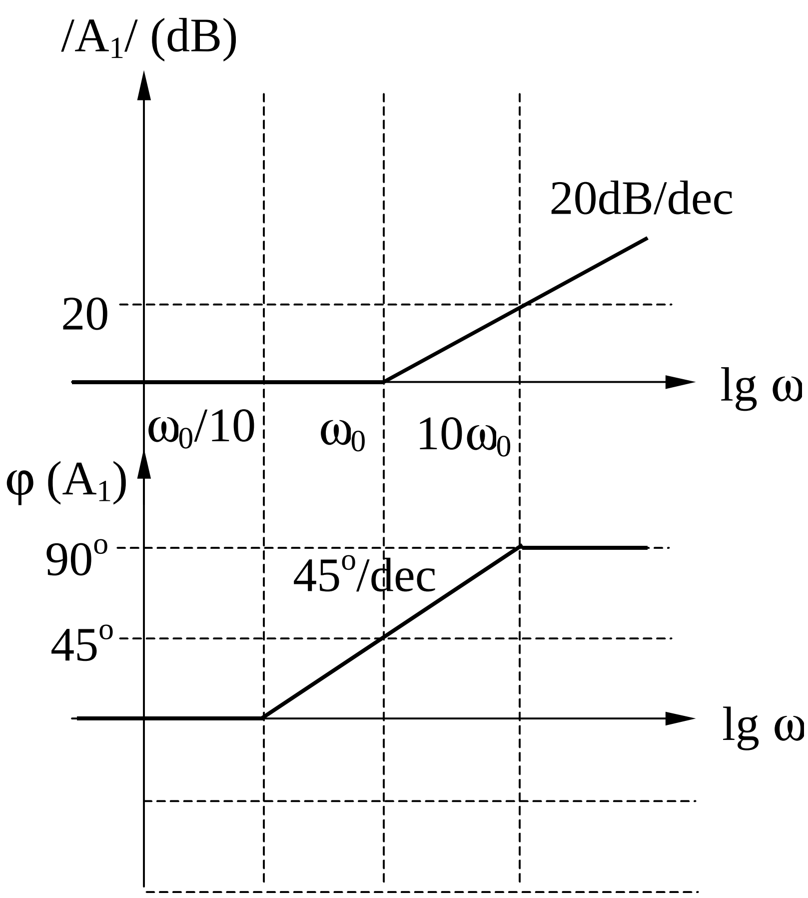


$$A_1 = 1 + j\frac{\omega}{\omega_0}$$

$$\left|A_1\right| = 20\,\mathbf{lg}\left[\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\varphi(A_1) = arctg\left(\frac{\omega}{\omega_0}\right)$$

Zero real negativ (A_1) - continuare



Amplitudine

$$|A_1| = 20\,lg\left[\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\omega \ll \omega_0 \Rightarrow |A_1| \to 0$$

(asimptota joasa frecventa)

$$\omega \gg \omega_0 \Rightarrow |A_1| \to 20\,lg\left(\frac{\omega}{\omega_0}\right)$$

(asimptota inalta frecventa)

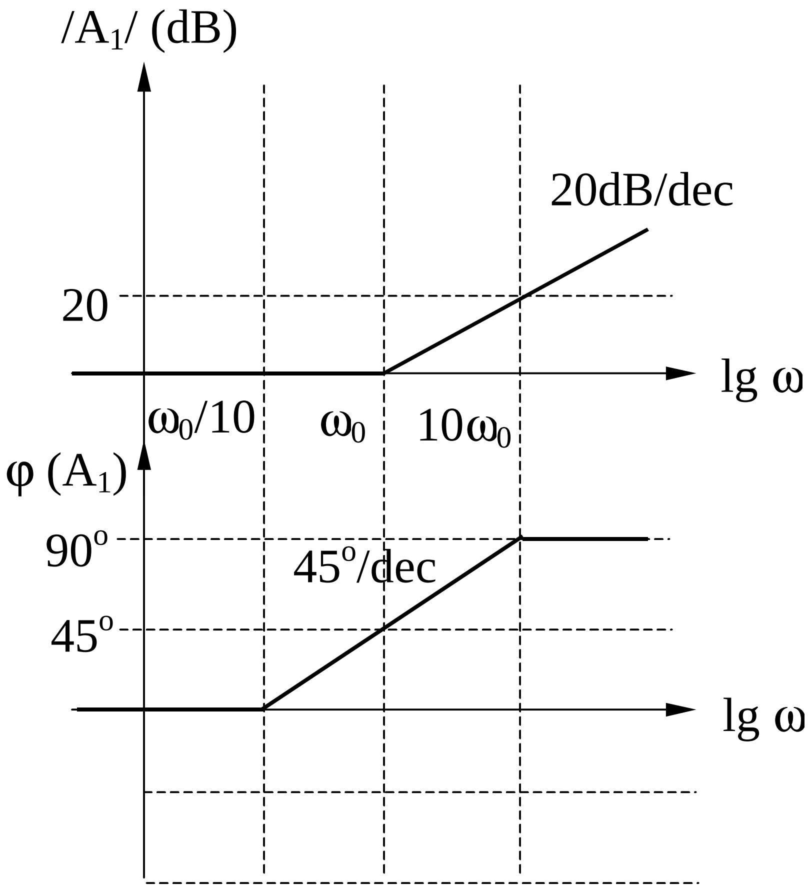
$$\omega = \omega_0 \Rightarrow |A_1| = 20\,lg\sqrt{1+1} = 3dB$$

$$\omega = \frac{\omega_0}{2} \Rightarrow |A_1| = 20\,lg\sqrt{1+\frac{1}{4}} = 1dB$$

$$\omega = 2\omega_0 \Rightarrow |A_1| = 20\,lg\sqrt{1+4} \cong 7\,dB$$

$$\Rightarrow \Delta|A_1| = (7-6)dB = 1dB$$

Zero real negativ (A_1) - continuare



Faza

$$\varphi(A_1) = arctg\left(\frac{\omega}{\omega_0}\right)$$

$$\omega << \omega_0 \Rightarrow \varphi(A_1) = arctg(0) = 0^o$$

(asimptota joasa frecventa)

$$\omega >> \omega_0 \Rightarrow \varphi(A_1) = arctg(\infty) = 90^o$$

(asimptota inalta frecventa)

$$\omega = \omega_0 \Rightarrow \varphi(A_1) = arctg(1) = 45^o$$

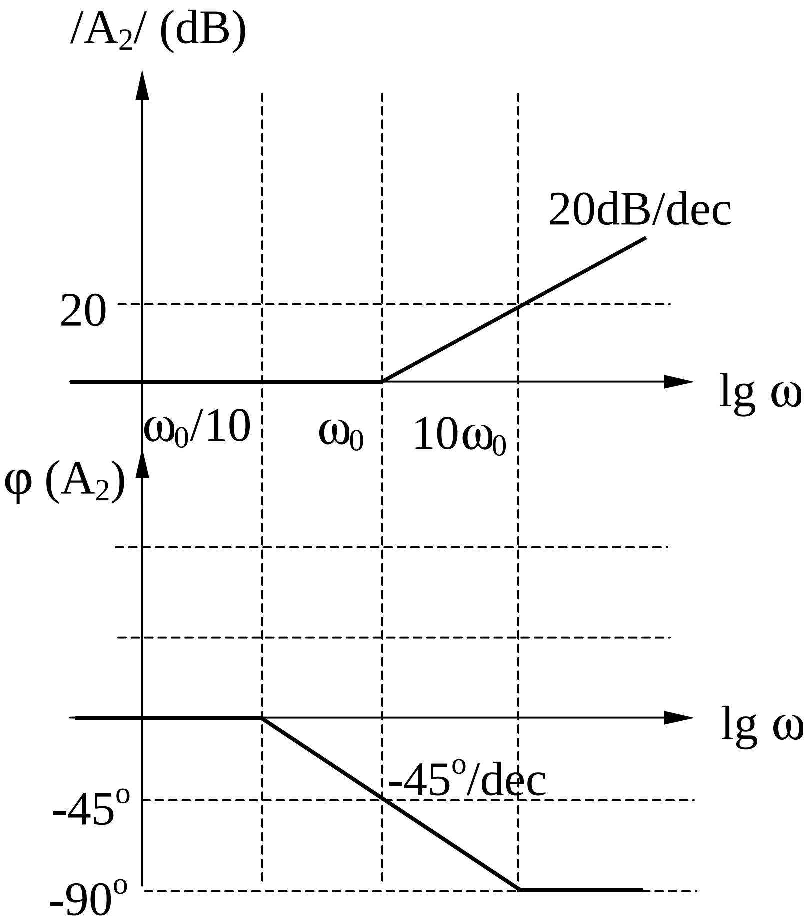
$$\omega = 10\omega_0 \Rightarrow \varphi(A_3) = arctg(10) \cong 84^o$$

(eroare 6^o)

$$\omega = \omega_0 / 10 \Rightarrow \varphi(A_3) = arctg(0{,}1) \cong 6^o$$

(eroare 6^o)

Zero real pozitiv (A_2)

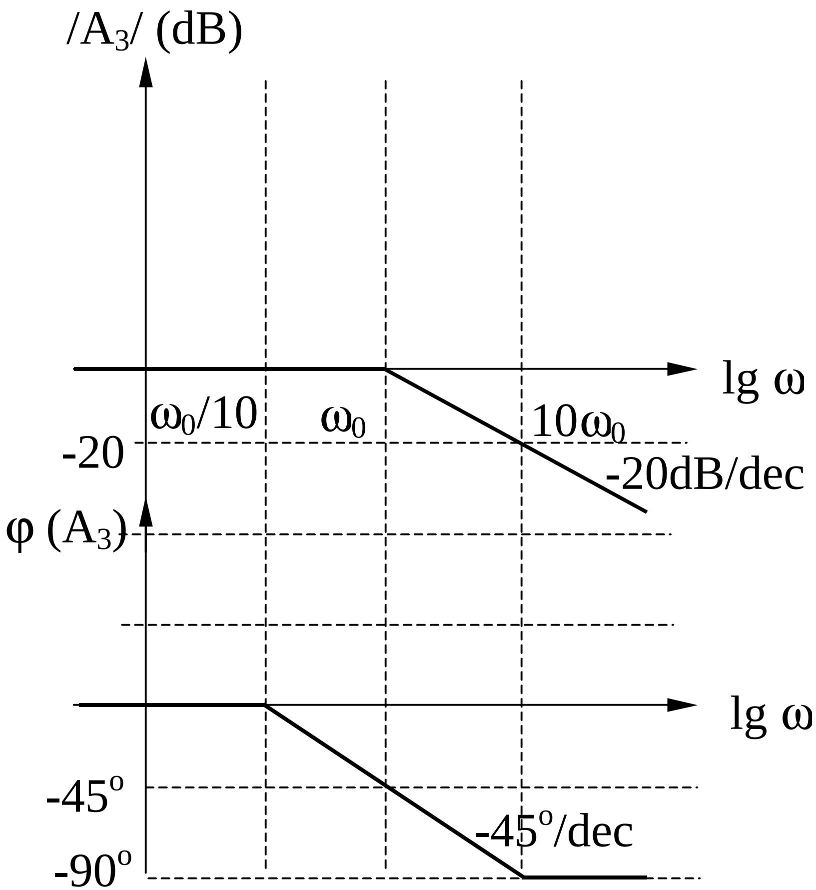


$$A_2 = 1 - j\frac{\omega}{\omega_0}$$

$$|A_2| = 20\,lg\left[\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\varphi(A_2) = -arctg\left(\frac{\omega}{\omega_0}\right)$$

Pol real negativ (A_3)

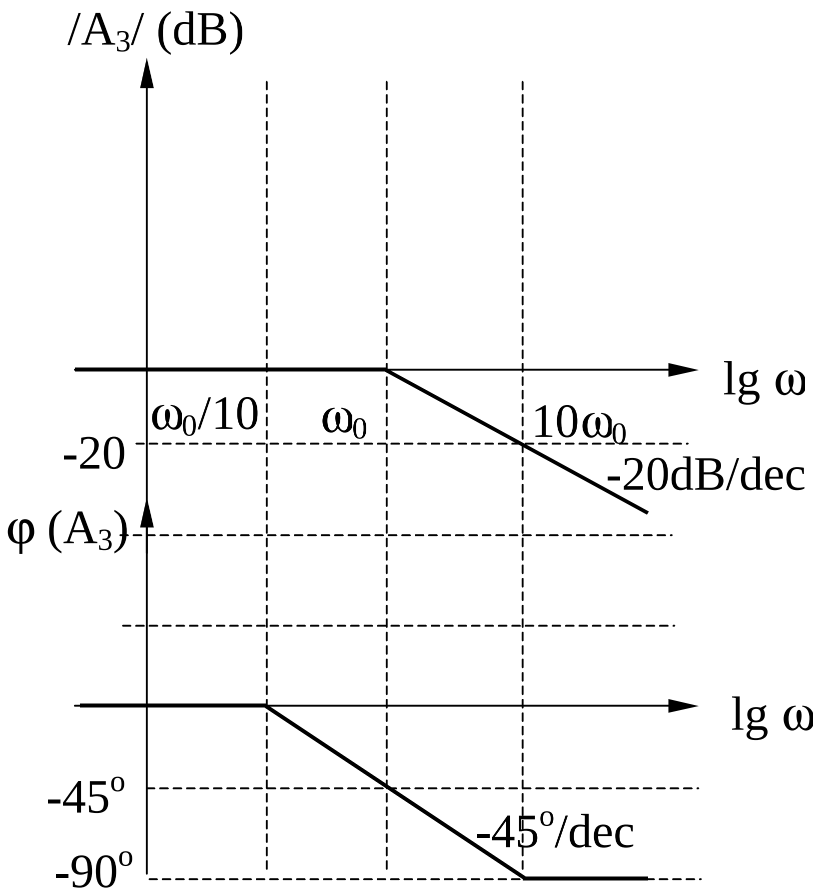


$$A_3 = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

$$|A_3| = -20\,lg\left[\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\varphi(A_3) = -arctg\left(\frac{\omega}{\omega_0}\right)$$

Pol real negativ (A_3) - continuare



Amplitudine

$$\left|A_3\right| = -20\,lg\left[\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\omega << \omega_0 \Rightarrow \left|A_3\right| \to 0$$

(asimptota joasa frecventa)

$$\omega >> \omega_0 \Rightarrow \left|A_3\right| \to -20\,lg\left(\frac{\omega}{\omega_0}\right)$$

(asimptota inalta frecventa)

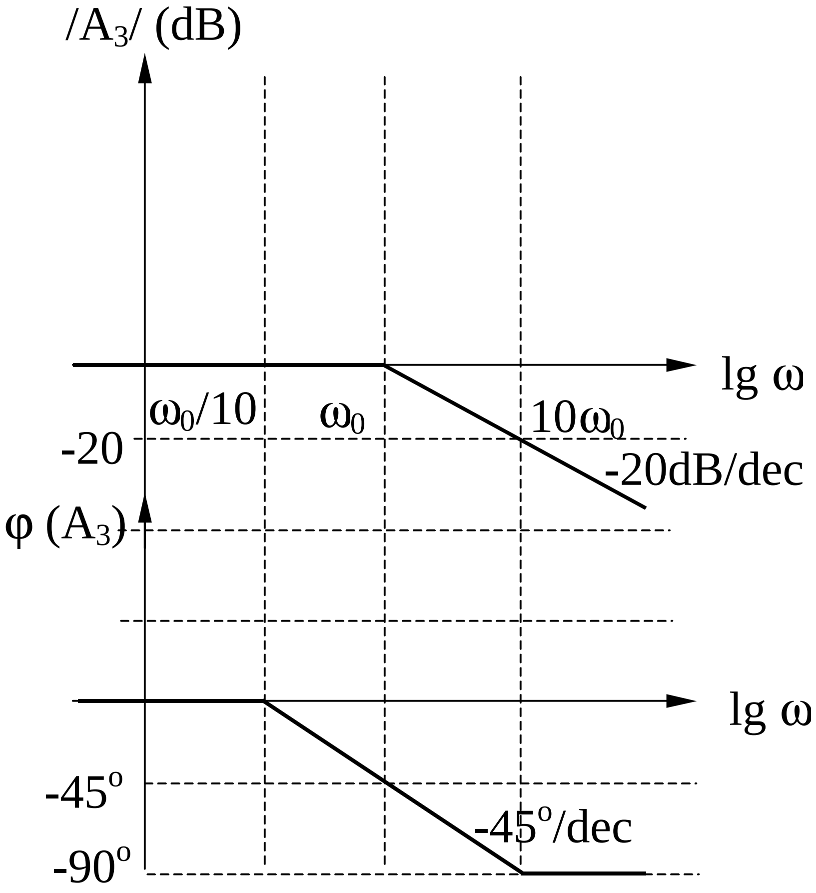
$$\omega = \omega_0 \Rightarrow \left|A_3\right| = -20\,lg\,\sqrt{1+1} = -3dB$$

$$\omega = \frac{\omega_0}{2} \Rightarrow \left|A_3\right| = -20\,lg\,\sqrt{1+\frac{1}{4}} = -1dB$$

$$\omega = 2\omega_0 \Rightarrow \left|A_3\right| = -20\,lg\,\sqrt{1+4} \cong -7\,dB$$

$$\Rightarrow \Delta\left|A_3\right| = \left(\left(-7\right)-\left(-6\right)\right)dB = -1dB$$

Pol real negativ (A_3) - continuare



Faza

$$\varphi(A_3) = -arctg\left(\frac{\omega}{\omega_0}\right)$$

$$\omega << \omega_0 \Rightarrow \varphi(A_3) = -arctg(0) = 0^o$$

(asimptota joasa frecventa)

$$\omega >> \omega_0 \Rightarrow \varphi(A_3) = -arctg(\infty) = -90^o$$

(asimptota inalta frecventa)

$$\omega = \omega_0 \Rightarrow \varphi(A_3) = -arctg(1) = -45^o$$

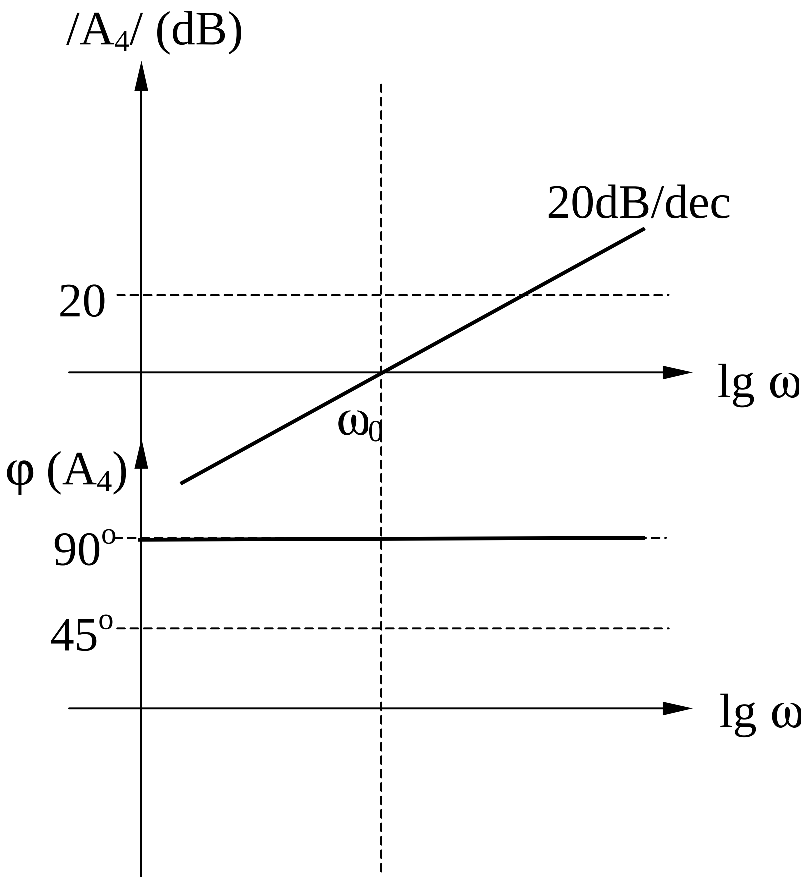
$$\omega = 10\omega_0 \Rightarrow \varphi(A_3) = -arctg(10) \cong -84^o$$

(eroare 6^o)

$$\omega = \omega_0 / 10 \Rightarrow \varphi(A_3) = -arctg(0,1) \cong -6^o$$

(eroare 6^o)

Zero simplu in origine (A_4)

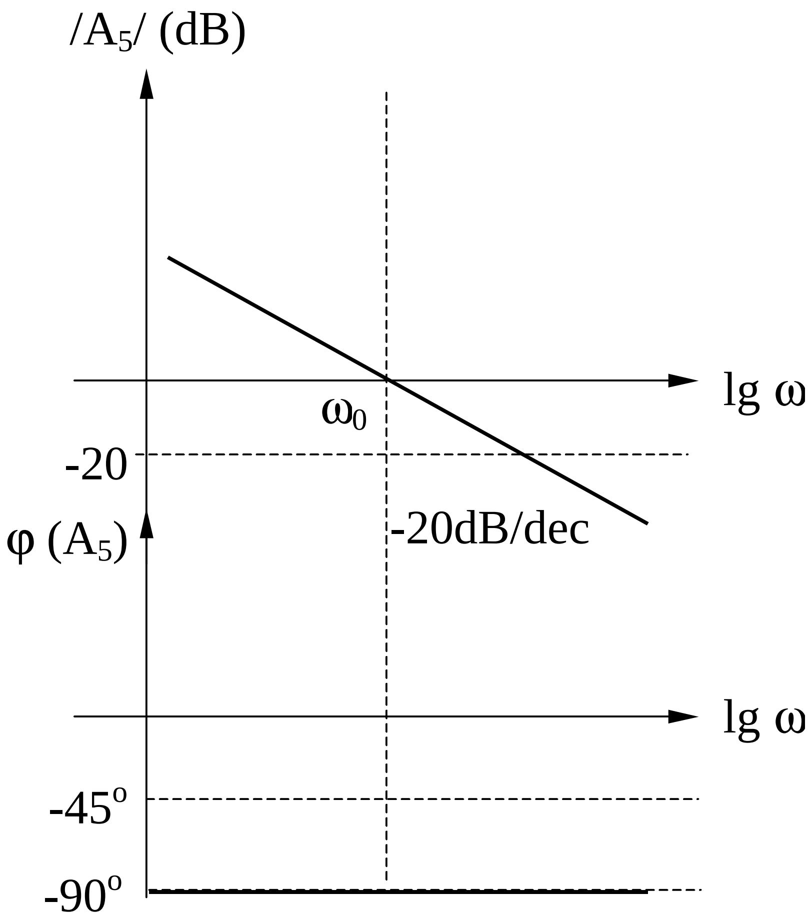


$$A_4 = j\frac{\omega}{\omega_0}$$

$$\left|A_4\right| = 20\,lg\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi\left(A_4\right) = 90^o$$

Pol simplu in origine (A_5)

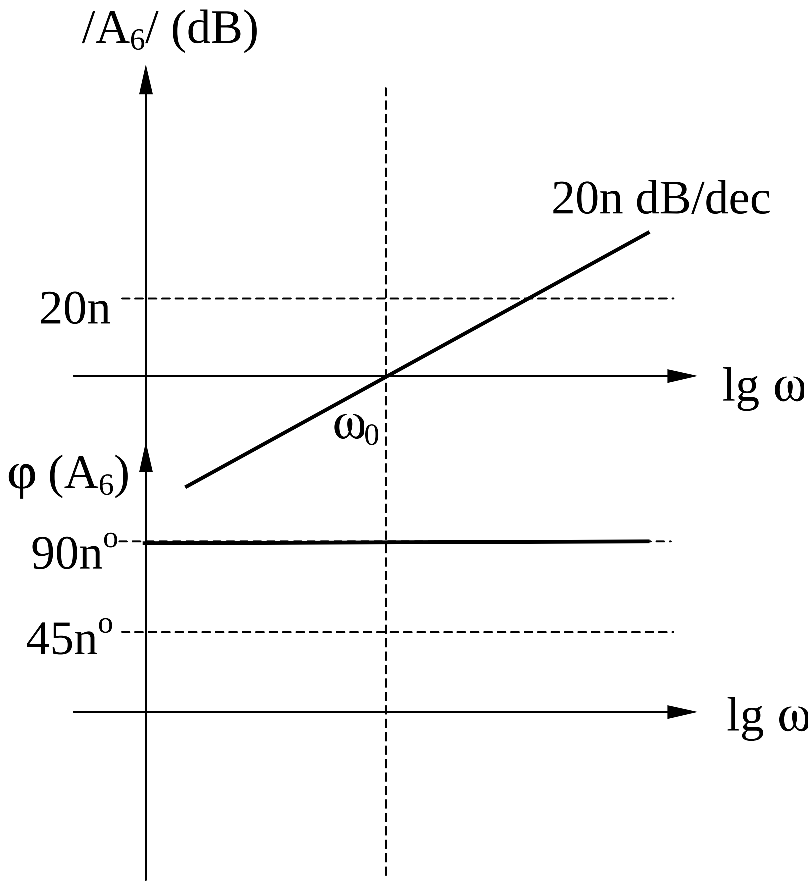


$$A_5 = \frac{1}{j\dfrac{\omega}{\omega_0}}$$

$$\left|A_5\right| = -20\,lg\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi\left(A_5\right) = -90^o$$

Zero multiplu in origine (A_6)

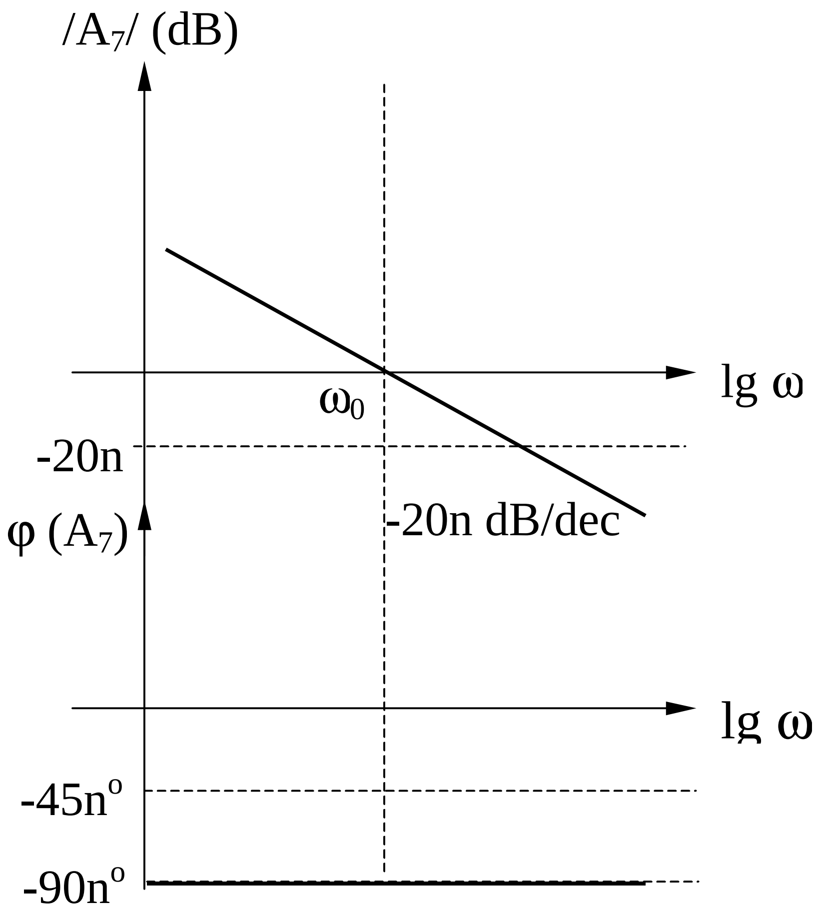


$$A_6 = \left(j \frac{\omega}{\omega_0} \right)^n$$

$$\left| A_6 \right| = 20 \times n \, lg \left(\frac{\omega}{\omega_0} \right)$$

$$\varphi\left(A_6 \right) = n \times 90^o$$

Pol multiplu in origine (A_7)

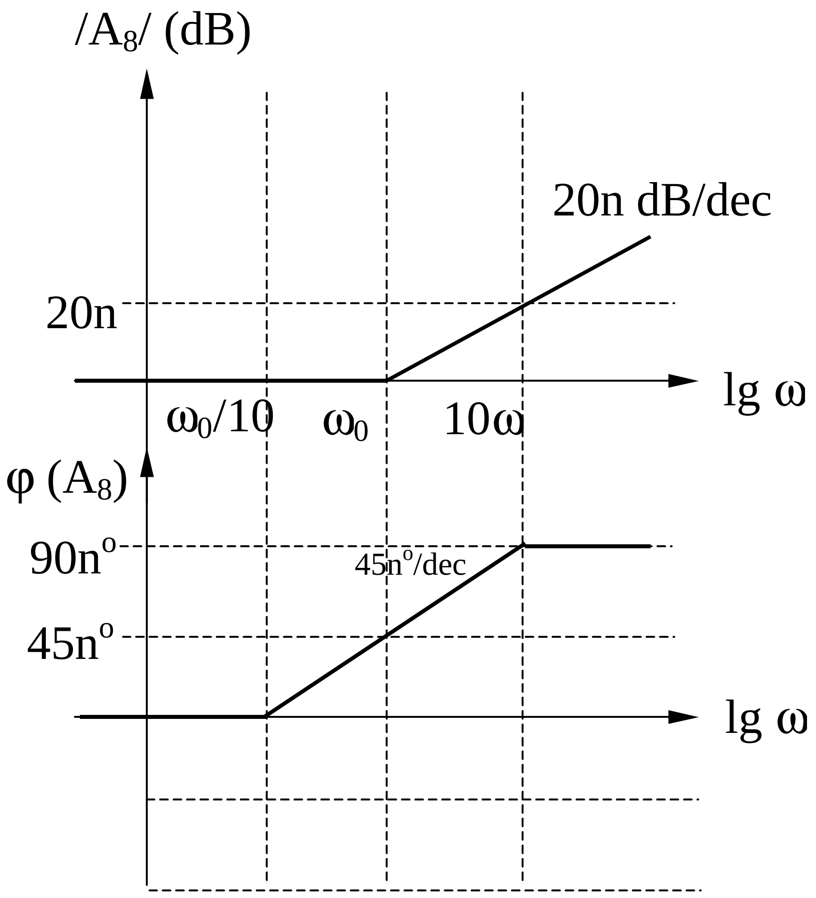


$$A_7 = \frac{1}{\left(j\frac{\omega}{\omega_0}\right)^n}$$

$$\left|A_7\right| = -20 \times n\,lg\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi\left(A_7\right) = -n \times 90^o$$

Zero real negativ multiplu (A_8)



$$A_8 = \left(1 + j\frac{\omega}{\omega_0}\right)^n$$

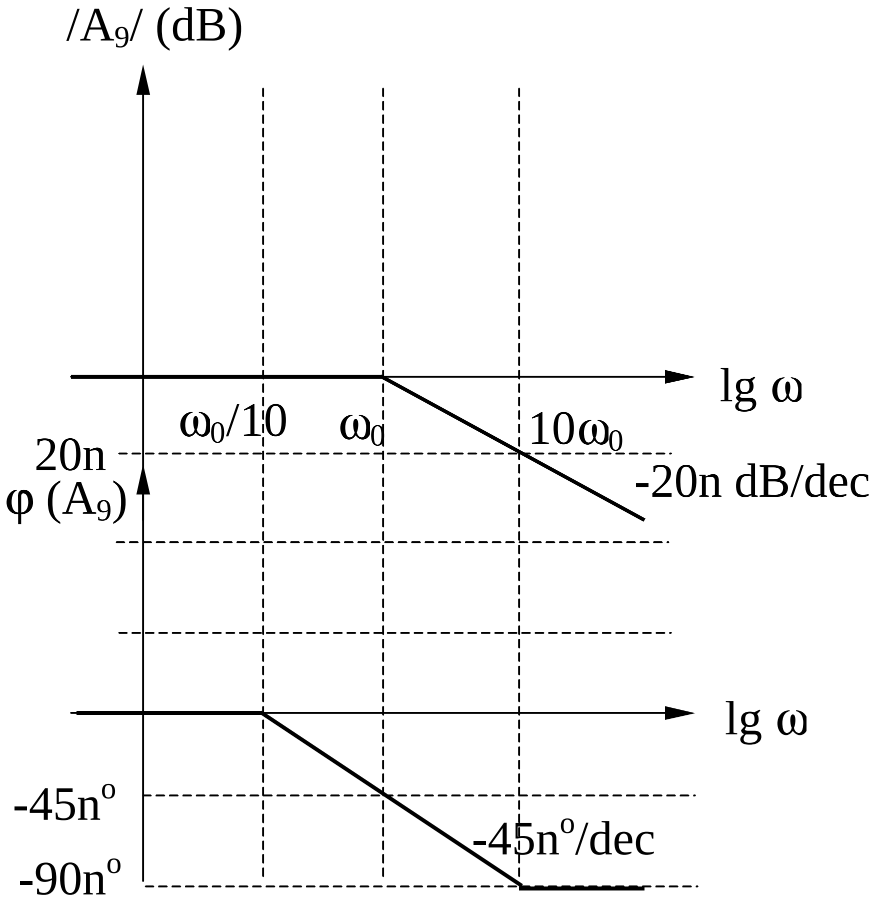
$$\left|A_8\right| = 20 \times n\, lg\left[\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\omega << \omega_0 \Rightarrow \left|A_8\right| \to 0$$

$$\omega >> \omega_0 \Rightarrow \left|A_8\right| \to 20 \times n\, lg\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi(A_8) = n \times arctg\left(\frac{\omega}{\omega_0}\right)$$

Pol real negativ multiplu (A_9)



$$A_9 = \frac{1}{\left(1 + j\frac{\omega}{\omega_0}\right)^n}$$

$$\left|A_9\right| = -20 \times n\,lg\left[\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\omega << \omega_0 \Rightarrow \left|A_9\right| \to 0$$

$$\omega >> \omega_0 \Rightarrow \left|A_9\right| \to -20 \times n\,lg\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi\left(A_9\right) = -n \times arctg\left(\frac{\omega}{\omega_0}\right)$$

Factor cuadratic (A_{10})

$$A_{10}(s) = \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{1}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

ω_0 = frecventa de rezonanta
ξ = factor de amortizare
Q = factor de calitate

$$\xi Q = \frac{1}{2}$$

Determinarea polilor: din ecuatia caracteristica:

$$s^2 + 2\xi\omega_0 s + \omega_0^2 = s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

Rezulta:

$$p_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

Factor cuadratic (A_{10}) - continuare

$$p_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

Situatii posibile:

1. Q < 0,5 (ξ > 1) $\Rightarrow$ 2 poli reali negativi
2. Q = 0,5 (ξ = 1) $\Rightarrow$ pol dublu

| - situatii deja analizate

3. Q > 0,5 (ξ < 1) $\Rightarrow$ 2 poli complex conjugati
4. Q $\rightarrow \infty$ ($\xi \rightarrow 0$) $\Rightarrow$ 2 poli imaginari

Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

$$p_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

$$Re\left(p_1, p_2\right) = -\xi\omega_0 = -\frac{\omega_0}{2Q}$$

$$Im\left(p_1, p_2\right) = \pm\omega_0\sqrt{1-\xi^2} = \pm\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

$$\left|p_1, p_2\right| = \omega_0$$

$$\varphi\left(p_1, p_2\right) = \frac{Im\left(p_1, p_2\right)}{Re\left(p_1, p_2\right)} = \pm arctg\sqrt{\frac{1}{\xi^2}-1} = \pm arctg\sqrt{4Q^2-1}$$

Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Caracteristica de frecventa

$$A_{10}(s) = \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{1}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$F(j\omega) = \omega_0^2 A_{10}(j\omega) = \frac{1}{1 + 2j\xi\frac{\omega}{\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2} = \frac{1}{1 + \frac{j}{Q}\frac{\omega}{\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$

Se noteaza: $u = \omega / \omega_0;\quad x = (\omega / \omega_0)^2$. Rezulta:

$$F(ju) = \frac{1}{1 + 2j\xi u - u^2} = \frac{1}{1 + \frac{j}{Q} u - u^2}$$

$$|F(ju)| = \frac{1}{\sqrt{\left(1 - u^2\right)^2 + 4\xi^2 u^2}}; \qquad |F(jx)| = \frac{1}{\sqrt{(1 - x)^2 + 4\xi^2 x}}$$

$$\varphi(ju) = -arctg\frac{2\xi u}{1 - u^2}$$

Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Caracteristica de frecventa (continuare)

$$\left|F(ju)\right|_{dB} = -20\lg\sqrt{\left(1-u^2\right)^2 + 4\xi^2 u^2}; \qquad \left|F(jx)\right|_{dB} = -20\lg\sqrt{(1-x)^2 + 4\xi^2 x}$$

Se noteaza: $f(x) = (1-x)^2 + 4\xi^2 x = 1 + \left(4\xi^2 - 2\right)x + x^2$

Conditia de minim a functiei f(x) se obtine prin anularea derivatei acesteia:

$$f'(x) = \left(4\xi^2 - 2\right) + 2x = 0 \Rightarrow x = 1 - 2\xi^2 > 0 \Rightarrow \xi < \frac{1}{\sqrt{2}} \cong 0{,}707$$

$$f''(x) = 2 > 0$$

Deci, functia f(x) va avea un minim pentru $x = 1-2\xi^2$ (echivalent cu **existenta unui maxim al $/F(jx)/_{dB}$) numai daca ξ < 0,707 (sau Q > 0,707)**.
Se va obtine un maxim pentru ω_P avand expresia:

$$x = 1 - 2\xi^2 \Leftrightarrow \omega_{Peak} = \omega_P = \omega_0\sqrt{1-2\xi^2} = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$$

Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Caracteristica de frecventa (continuare)

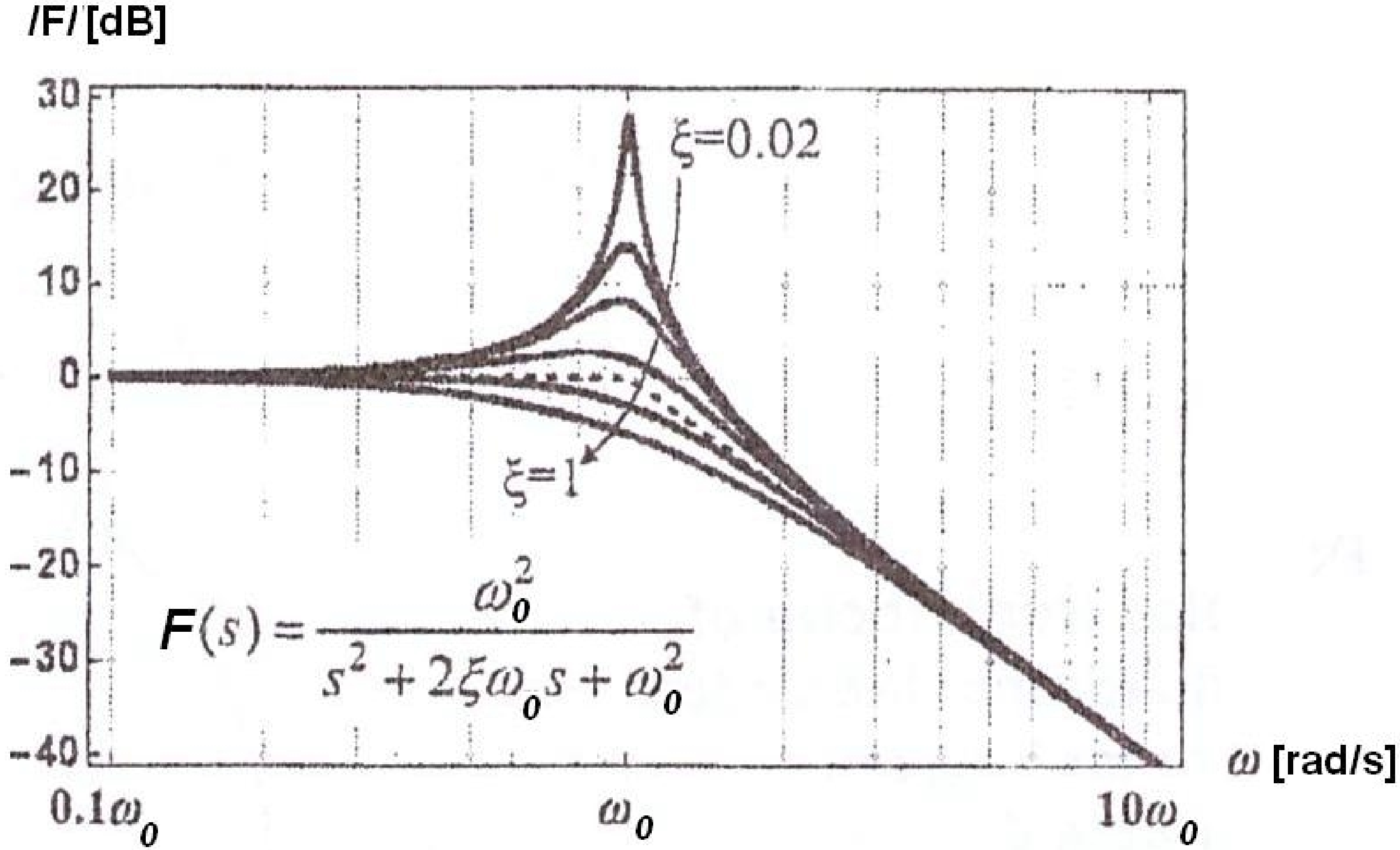
Valoarea acestui maxim este:

$$\left|F(jx)\right|_{dB} = -20\,lg\sqrt{\left(1-1+2\xi^2\right)^2+4\xi^2\left(1-2\xi^2\right)}$$

$$\left|F(jx)\right|_{dB} = 20\,lg\,\frac{1}{2\xi\sqrt{1-\xi^2}} = 20\,lg\,\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}}$$

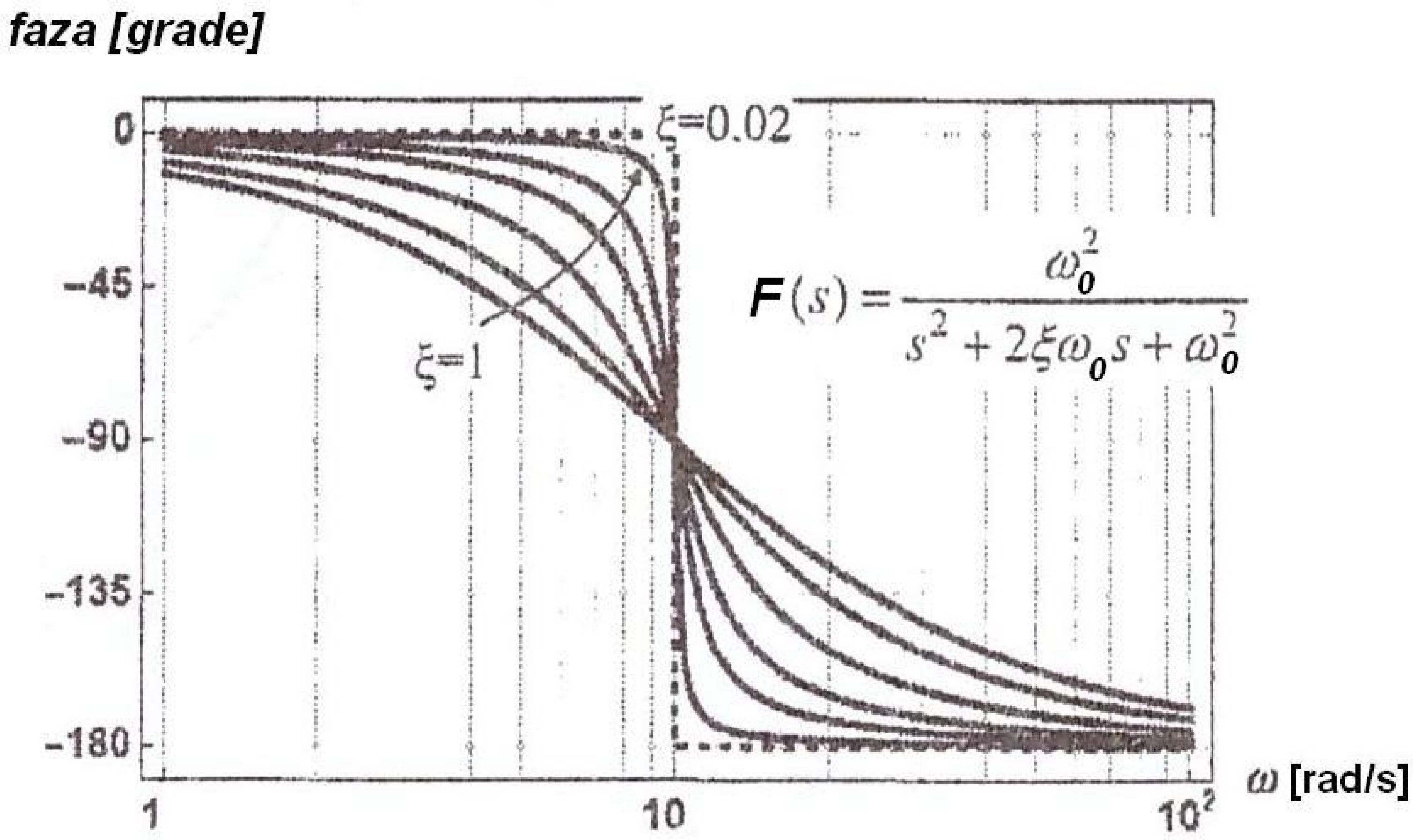
Factor cuadratic (A_{10}) – poli complex conjugati ($Q > 0,5$)

Caracteristica de frecventa (continuare)



Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Caracteristica de frecventa (continuare)



Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Trasarea diagramelor Bode (MODUL) pentru polii complex conjugati

$$\left|F(ju)\right|_{dB} = -20\,lg\sqrt{\left(1-u^2\right)^2 + 4\xi^2 u^2}$$

$$\left|F(j\omega)\right|_{dB} = -20\,lg\sqrt{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2}$$

$\omega << \omega_0 \Rightarrow \left|F(j\omega)\right|_{dB} \to 0dB$ (asimptota joasa frecventa)

$\omega >> \omega_0 \Rightarrow \left|F(j\omega)\right|_{dB} \cong -40\,lg\left(\frac{\omega}{\omega_0}\right)$ (asimptota inalta frecventa)

$$\omega = \omega_0 \Rightarrow \left|F(j\omega)\right|_{dB} = -20\,lg(2\xi)$$

Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Trasarea diagramelor Bode (MODUL) pentru polii complex conjugati (continuare)
(daca EXISTA maxim: $0 < \xi < 0,707$ sau $Q > 0,707$)

1. Se traseaza asimptotele:
   - la JF: 0dB pana in ω_0
   - la IF: o dreapta cu panta de -40dB/decada pornind din ω_0
2. Se calculeaza $\omega = \omega_P$ pentru care se obtine maximul:

$$\omega_P = \omega_0\sqrt{1-2\xi^2} = \omega_0\sqrt{1-\frac{1}{2Q^2}}$$

3. Se calculeaza valoarea maximului:

$$\left|F_P\right|_{dB} = 20\lg\frac{1}{2\xi\sqrt{1-\xi^2}} = 20\lg\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}}$$

4. Se traseaza o curba care tinde asimptotic catre cele 2 axe (JF si IF) si care trece prin punctul de maxim (ω_P, $/F_P/_{dB}$)

Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Trasarea diagramelor Bode (MODUL) pentru polii complex conjugati (continuare)
(daca NU EXISTA maxim: $\xi > 0,707$ sau $Q < 0,707$)

1. Se traseaza asimptotele:
   - la JF: 0dB pana in ω_0
   - la IF: o dreapta cu panta de -40dB/decada pornind din ω_0

2. Se traseaza o curba care tinde asimptotic catre cele 2 axe (JF si IF)

Factor cuadratic (A_{10}) – poli complex conjugati (Q > 0,5)

Trasarea diagramelor Bode (FAZA) pentru polii complex conjugati

$$\varphi(ju) = -arctg\,\frac{2\xi u}{1-u^2} = -arctg\,\frac{\frac{1}{Q}u}{1-u^2}$$

$$\varphi(j\omega) = -arctg\,\frac{2\xi\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2} = -arctg\,\frac{\frac{1}{Q}\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}$$

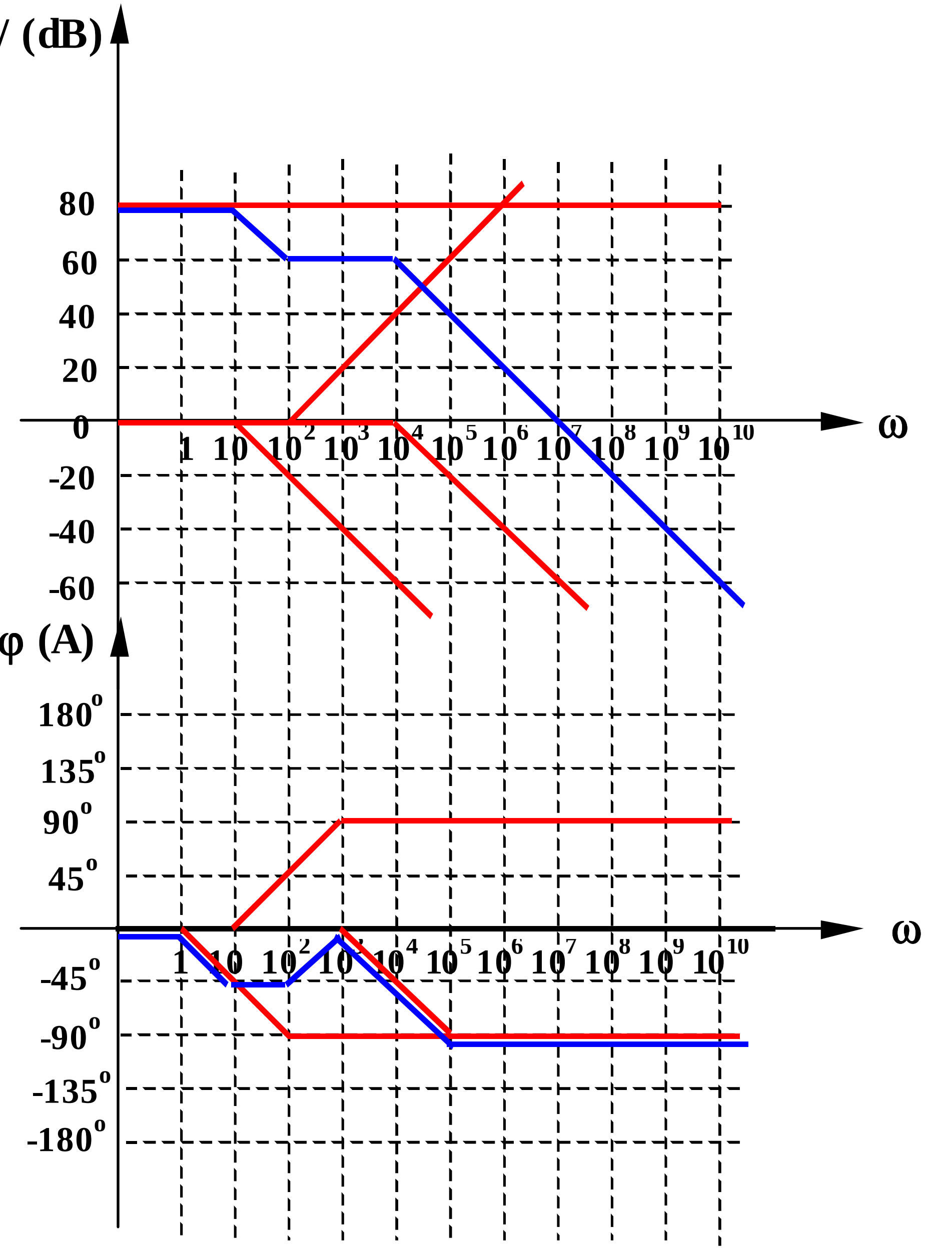
$\omega \ll \omega_0 \Rightarrow \varphi \to -arctg\,0 = 0$ (asimptota joasa frecventa)

$\omega \gg \omega_0 \Rightarrow \varphi \to -arctg\,0 = -180^o$ (asimptota inalta frecventa)

$\omega = \omega_0 \Rightarrow \varphi = arctg(\infty) = -90^o$

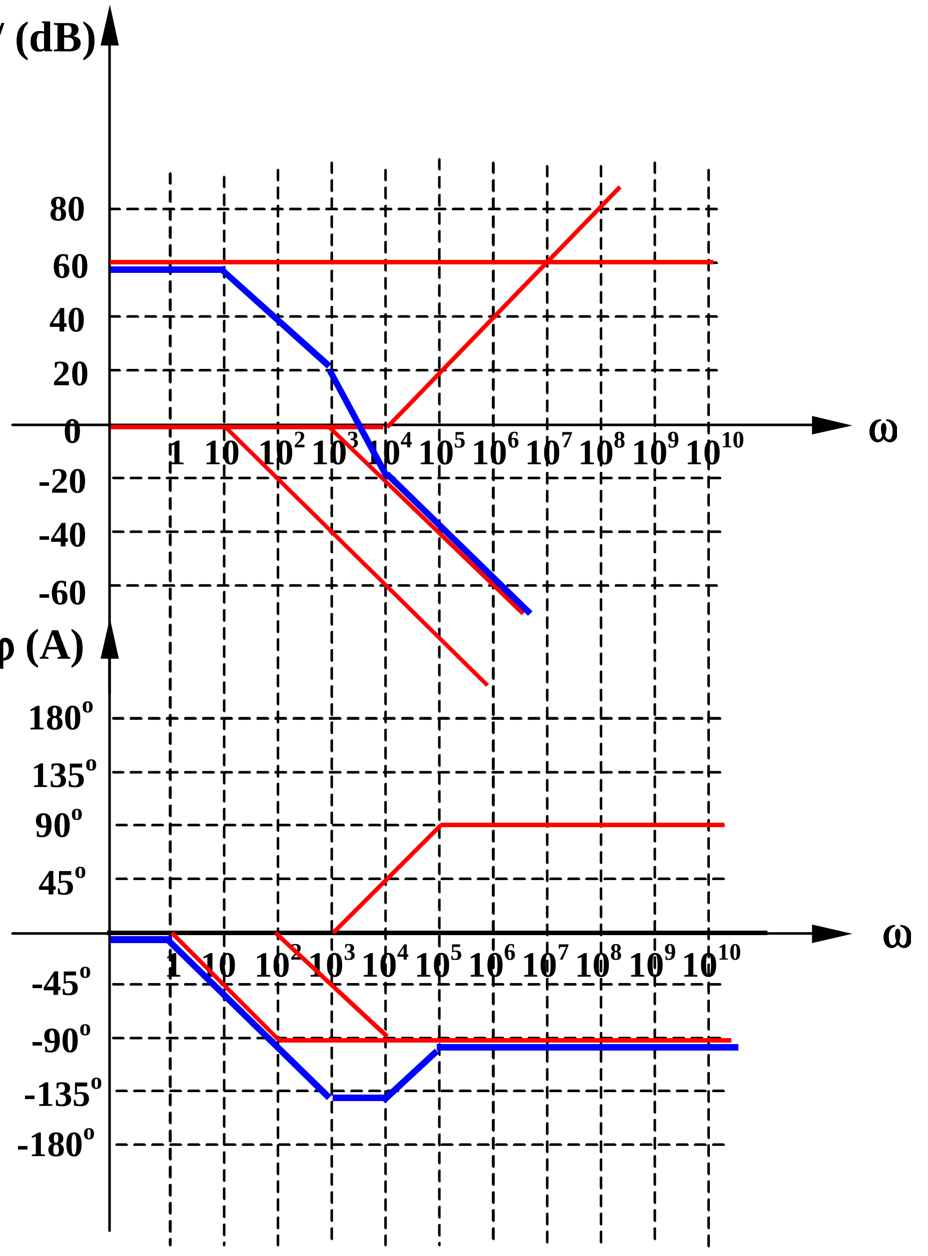
Exemplul 1

$$A(j\omega) = 10^4 \frac{1 + j\dfrac{\omega}{10^2}}{\left(1 + j\dfrac{\omega}{10}\right)\left(1 + j\dfrac{\omega}{10^4}\right)}$$



Exemplul 2

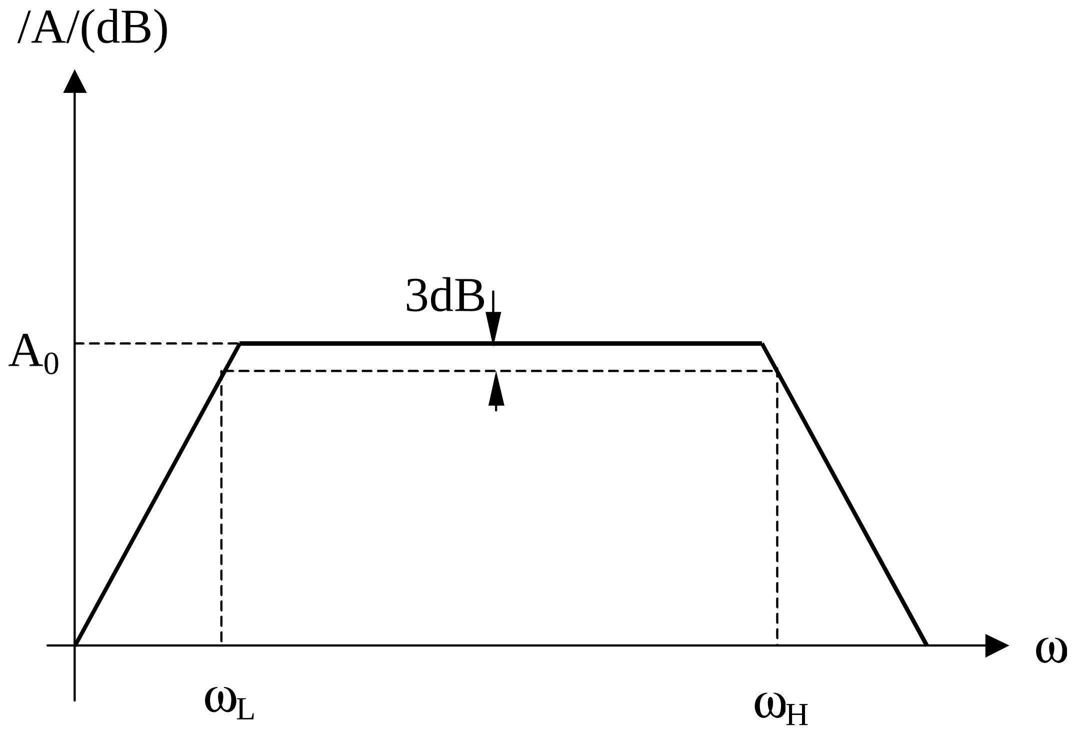
$$A(j\omega) = 10^3 \frac{1 + j\dfrac{\omega}{10^4}}{\left(1 + j\dfrac{\omega}{10}\right)\left(1 + j\dfrac{\omega}{10^3}\right)}$$



7.2. Raspunsul in frecventa al amplificatoarelor

7.2. Raspunsul in frecventa al amplificatoarelor

7.2.1. Banda de frecventa



7.2. Raspunsul in frecventa al amplificatoarelor

7.2.1. Banda de frecventa

La frecvente medii:
- condensatoarele de cuplaj si de decuplare sunt scurt-circuite
- condensatoarele interne ale dispozitivelor sunt circuite deschise

La frecvente joase:
- condensatoarele de cuplaj si de decuplare nu mai sunt scurt-circuite
- condensatoarele interne ale dispozitivelor sunt circuite deschise

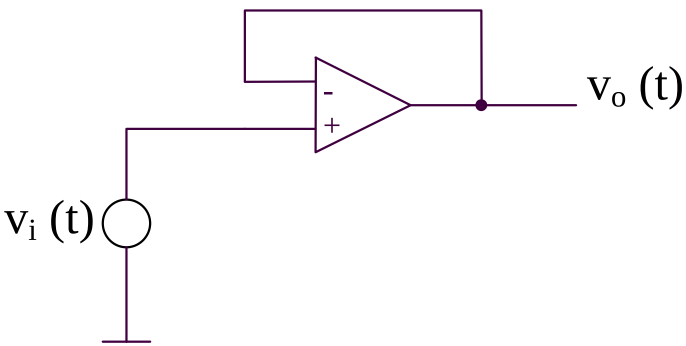
La frecvente inalte:
- condensatoarele de cuplaj si de decuplare sunt scurt-circuite
- condensatoarele interne ale dispozitivelor nu mai sunt circuite deschise

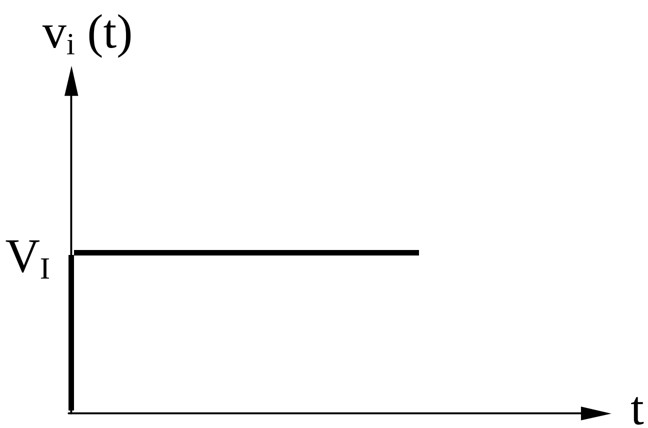
7.2. Raspunsul in frecventa al amplificatoarelor

7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Slew-Rate-ul (SR) reprezinta viteza maxima de crestere a tensiunii de iesire a unui amplificator operational in CONDITII DE SEMNAL MARE.

Evaluarea performantei la semnal mare si inalta frecventa pentru un AO





Amplificarea in bucla inchisa are expresia:

$$A = \frac{a}{1+af}$$

Se considera un AO cu un singur pol:

$$a = \frac{a_0}{1+\frac{s}{\omega_H}}$$

Circuitul fiind repetor, $f = 1$. Se obtine:

$$A(s) = \frac{V_o(s)}{V_i(s)} = \frac{a(s)}{1+a(s)} = \frac{1}{1+\frac{1}{a(s)}} \cong \frac{1}{1+\frac{s}{\omega_u}}$$

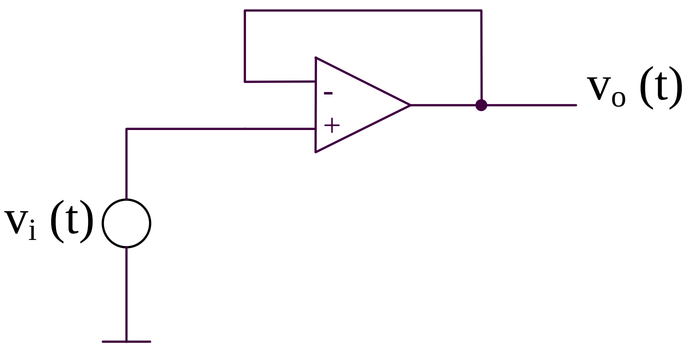
unde:

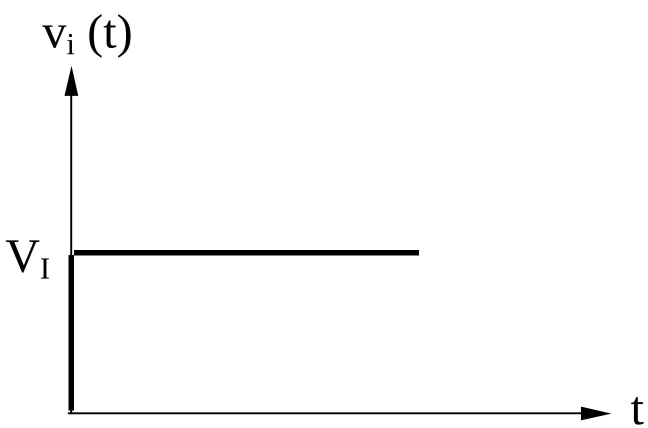
$$\omega_u = a_0 \omega_H$$

7.2. Raspunsul in frecventa al amplificatoarelor

7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Evaluarea performantei la semnal mare si inalta frecventa pentru un AO





Semnalul de intrare are transformata Laplace:

$$V_i(s) = \frac{V_I}{s}$$

Rezulta:

$$V_o(s) = V_i(s)A(s) = \frac{V_I}{s}\frac{1}{1+\frac{s}{\omega_u}} =$$

$$= V_I\left(\frac{1}{s} - \frac{1}{s+\omega_u}\right)$$

Deci:

$$v_O(t) = V_I\left(1 - e^{-\omega_u t}\right)$$

7.2. Raspunsul in frecventa al amplificatoarelor

7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Evaluarea performantei la semnal mare si inalta frecventa pentru un AO

$$V_I\left(1-e^{-\omega_u t_{90\%}}\right)=0,9V_I \Rightarrow t_{90\%}=-\frac{ln(0,1)}{\omega_u}$$

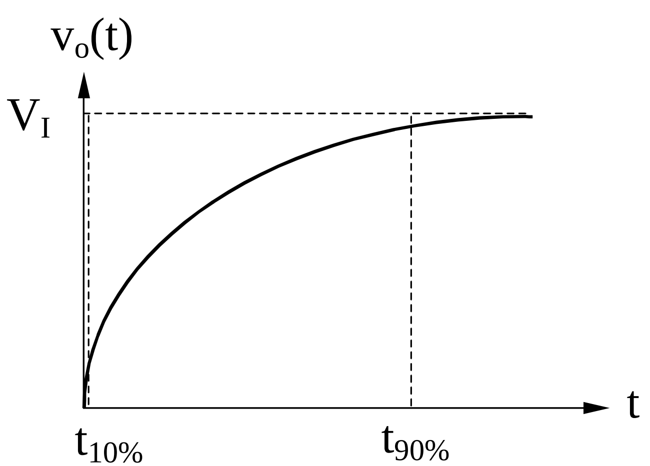
$$V_I\left(1-e^{-\omega_u t_{10\%}}\right)=0,1V_I \Rightarrow t_{10\%}=-\frac{ln(0,9)}{\omega_u}$$

$$t_r=t_{90\%}-t_{10\%}=\frac{ln(9)}{\omega_u}\cong\frac{2,2}{\omega_u}$$

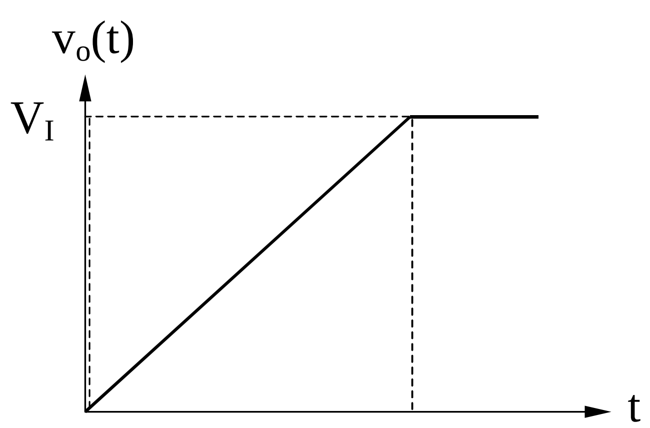
Rezulta:

$$t_r\omega_u=2,2 \quad \text{sau} \quad t_r f_u=0,35$$

Raspuns in timp estimat



Raspuns in timp masurat



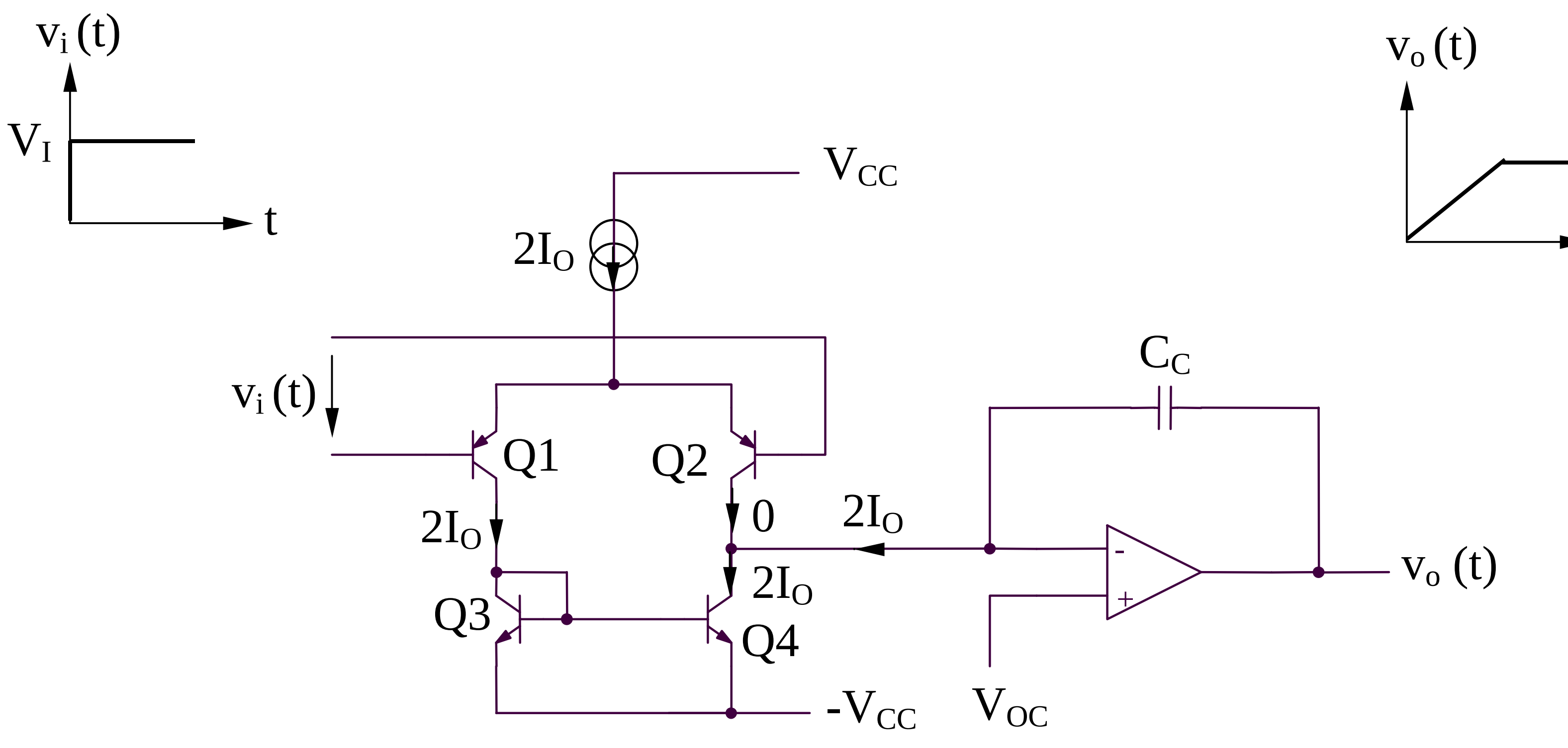
7.2. Raspunsul in frecventa al amplificatoarelor

7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Evaluarea performantei la semnal mare si inalta frecventa pentru un AO

Diferenta majora intre cele doua raspunsuri in timp este cauzata de faptul ca analiza de SEMNAL MIC nu poate fi utilizata pentru a determina comportamentul circuitului in conditii de SEMNAL MARE.

Structura tipica a unui AO este:



7.2. Raspunsul in frecventa al amplificatoarelor

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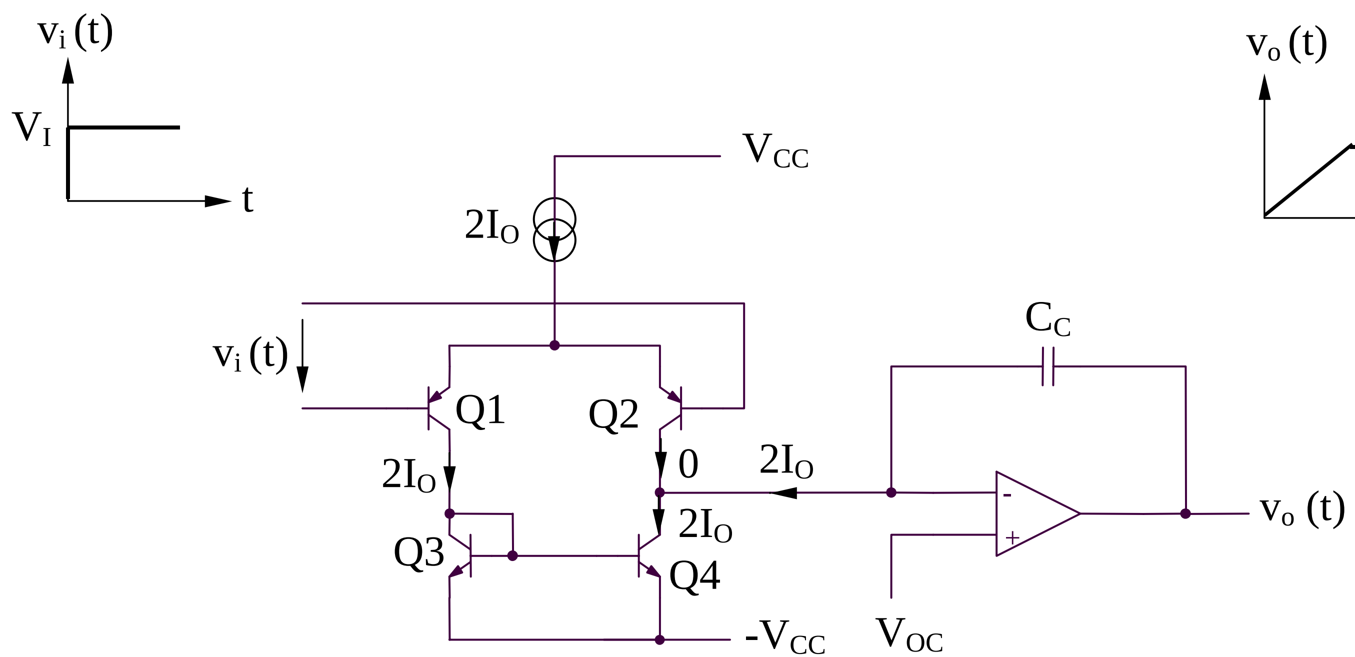

La t = 0, semnalul de intrare creste de la 0 la V_I (de ordinul voltilor), dar tensiunea de iesire nu raspunde instantaneu. Tensiunea mare de intrare aplicata AO va scoate complet AD de intrare din zona liniara. Prin urmare, $i_{C2} = 0$, iar $i_{C1} = i_{C3} = i_{C4} = 2I_O$, curent ce va incarca condensatorul C_C la o tensiune v_O, cu o panta $dv_O/dt = SR$:

$$v_O = v_{C_C} = \frac{1}{C_C}\int_0^t 2I_O dt \qquad \frac{dv_O}{dt} = \boxed{\frac{2I_O}{C_C} = SR} = constant$$

7.2. Raspunsul in frecventa al amplificatoarelor

7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Efectul limitarilor introduse de SR asupra functionarii la semnal mare de intrare de tip sinusoidal

Ideal, tensiunea de iesire va urmari tensiunea de intrare:

$$v_o = V_o \sin \omega t$$

cu o viteza de variatie maxima a tensiunii de iesire avand expresia:

$$\left.\frac{dv_o}{dt}\right|_{max} = \omega V_o$$

a. Daca $\left.\frac{dv_o}{dt}\right|_{max} < SR$, v_o va urmari perfect tensiunea de intrare

b. Daca $\left.\frac{dv_o}{dt}\right|_{max} > SR$, v_o va fi afectata de distorsiuni puternice

Frecventa maxima a semnalului de iesire de amplitudine maxima (aproximativ egala cu tensiunea de alimentare) nedistorsionat, f_{max}, se poate determina astfel:

$$\left.\frac{dv_o}{dt}\right|_{max} = \omega_{max} V_{OM} = SR \Rightarrow f_{max} = \frac{SR}{2\pi V_{OM}}$$