

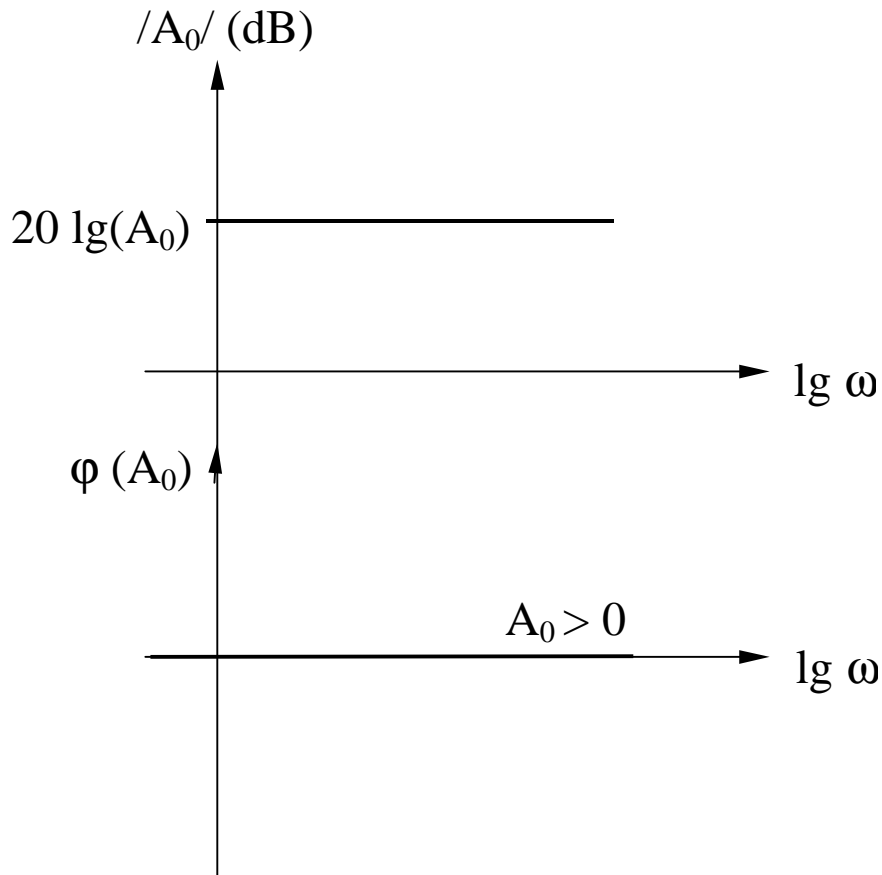
# **Capitolul 7**

## **Raspunsul in frecventa al circuitelor. Stabilitatea circuitelor cu reactie**

## **7.1. Caracteristicile de frecventa ale functiilor elementare. Diagrame Bode**

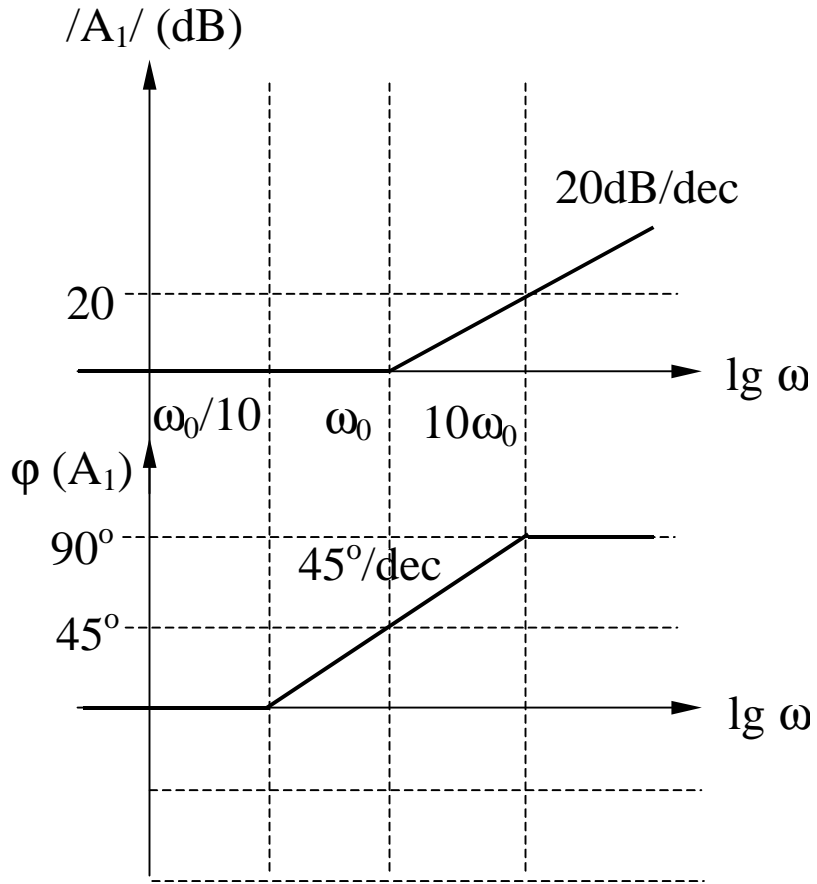
# 7.1. Caracteristicile de frecventa ale functiilor elementare. Diagrame Bode

## Constanta ( $A_0$ )



$$A_0 = ct.$$

## Zero real negativ ( $A_1$ )



$$A_1 = 1 + j \frac{\omega}{\omega_0}$$

$$|A_1| = 20 \lg \left[ \sqrt{1 + \left( \frac{\omega}{\omega_0} \right)^2} \right]$$

$$\varphi(A_1) = \arctg \left( \frac{\omega}{\omega_0} \right)$$

# Zero real negativ ( $A_1$ ) - continuare

## Amplitudine

$$|A_1| = 20 \lg \left[ \sqrt{1 + \left( \frac{\omega}{\omega_0} \right)^2} \right]$$

$$\omega \ll \omega_0 \Rightarrow |A_1| \rightarrow 0$$

(asimptota joasa frecventa)

$$\omega \gg \omega_0 \Rightarrow |A_1| \rightarrow 20 \lg \left( \frac{\omega}{\omega_0} \right)$$

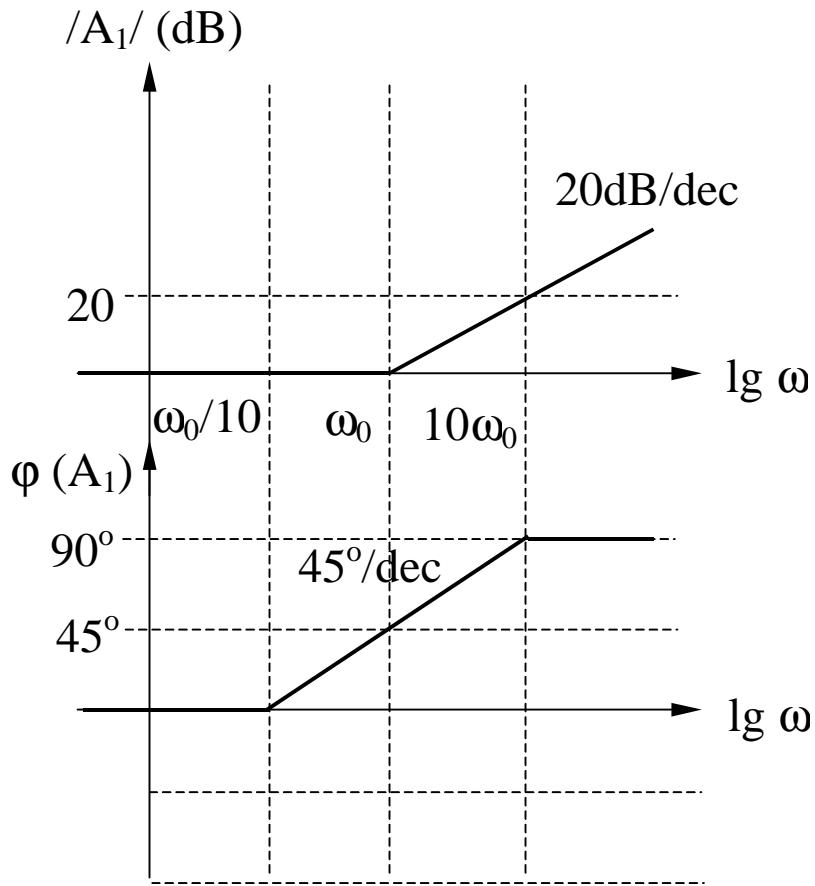
(asimptota inalta frecventa)

$$\omega = \omega_0 \Rightarrow |A_1| = 20 \lg \sqrt{1+1} = 3dB$$

$$\omega = \frac{\omega_0}{2} \Rightarrow |A_1| = 20 \lg \sqrt{1 + \frac{1}{4}} = 1dB$$

$$\omega = 2\omega_0 \Rightarrow |A_1| = 20 \lg \sqrt{1+4} \cong 7dB$$

$$\Rightarrow \Delta|A_1| = (7 - 6)dB = 1dB$$



## Zero real negativ ( $A_1$ ) - continuare

### Faza

$$\varphi(A_1) = \arctg\left(\frac{\omega}{\omega_0}\right)$$

$$\omega \ll \omega_0 \Rightarrow \varphi(A_1) = \arctg(0) = 0^\circ$$

(asimptota joasa frecventa)

$$\omega \gg \omega_0 \Rightarrow \varphi(A_1) = \arctg(\infty) = 90^\circ$$

(asimptota inalta frecventa)

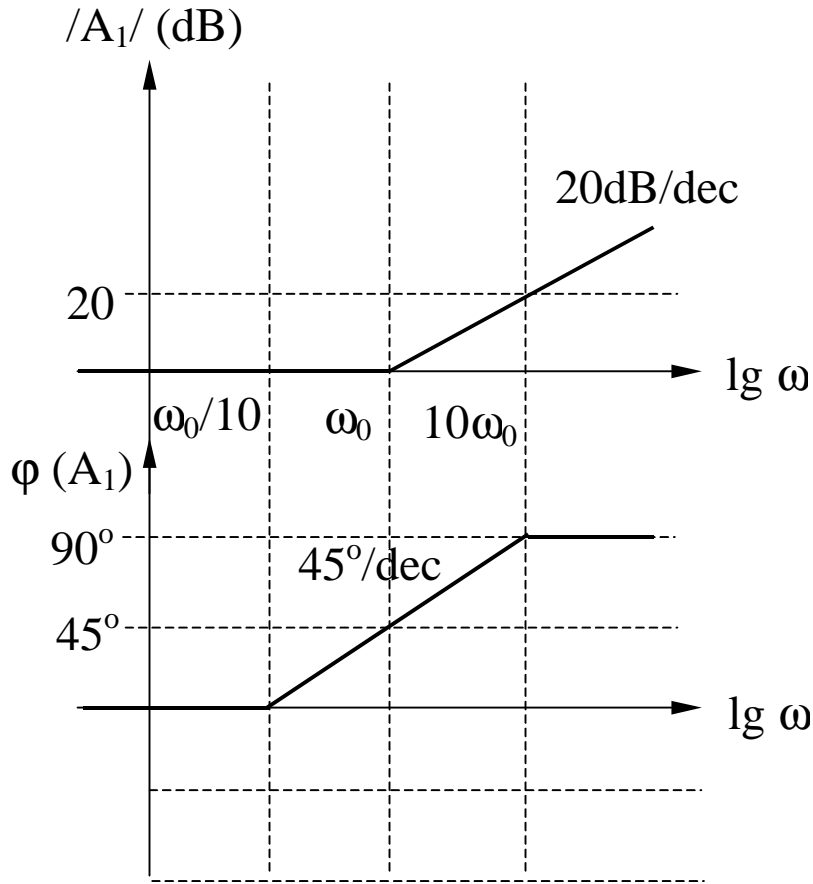
$$\omega = \omega_0 \Rightarrow \varphi(A_1) = \arctg(1) = 45^\circ$$

$$\omega = 10\omega_0 \Rightarrow \varphi(A_3) = \arctg(10) \cong 84^\circ$$

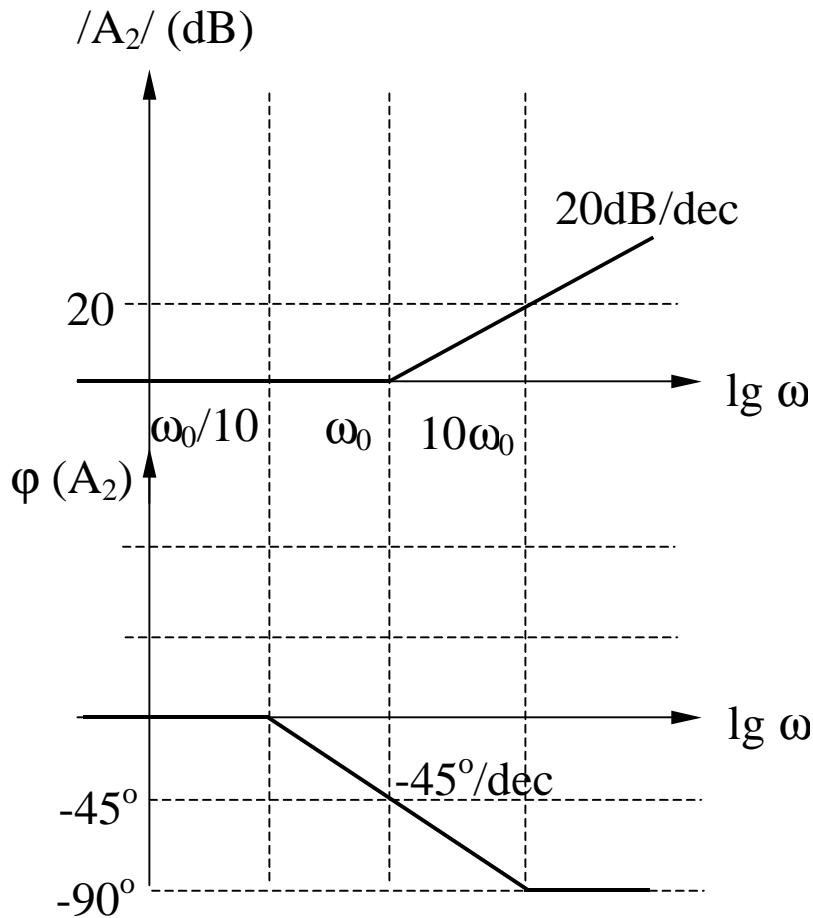
(eroare  $6^\circ$ )

$$\omega = \omega_0 / 10 \Rightarrow \varphi(A_3) = \arctg(0,1) \cong 6^\circ$$

(eroare  $6^\circ$ )



## Zero real pozitiv ( $A_2$ )

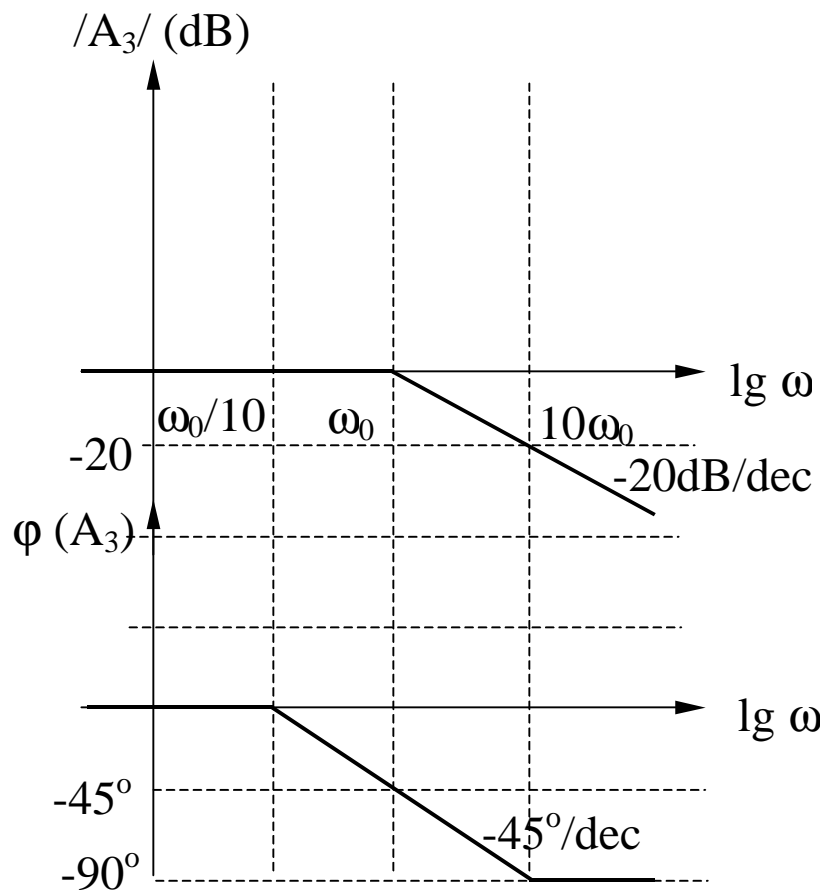


$$A_2 = 1 - j \frac{\omega}{\omega_0}$$

$$|A_2| = 20 \lg \left[ \sqrt{1 + \left( \frac{\omega}{\omega_0} \right)^2} \right]$$

$$\varphi(A_2) = -\arctg \left( \frac{\omega}{\omega_0} \right)$$

## Pol real negativ ( $A_3$ )



$$A_3 = \frac{1}{1 + j \frac{\omega}{\omega_0}}$$

$$|A_3| = -20 \lg \left[ \sqrt{1 + \left( \frac{\omega}{\omega_0} \right)^2} \right]$$

$$\varphi(A_3) = -\text{arctg} \left( \frac{\omega}{\omega_0} \right)$$



# Pol real negativ ( $A_3$ ) - continuare

## Amplitudine

$$|A_3| = -20 \lg \left[ \sqrt{1 + \left( \frac{\omega}{\omega_0} \right)^2} \right]$$

$$\omega \ll \omega_0 \Rightarrow |A_3| \rightarrow 0$$

(asimptota joasa frecventa)

$$\omega \gg \omega_0 \Rightarrow |A_3| \rightarrow -20 \lg \left( \frac{\omega}{\omega_0} \right)$$

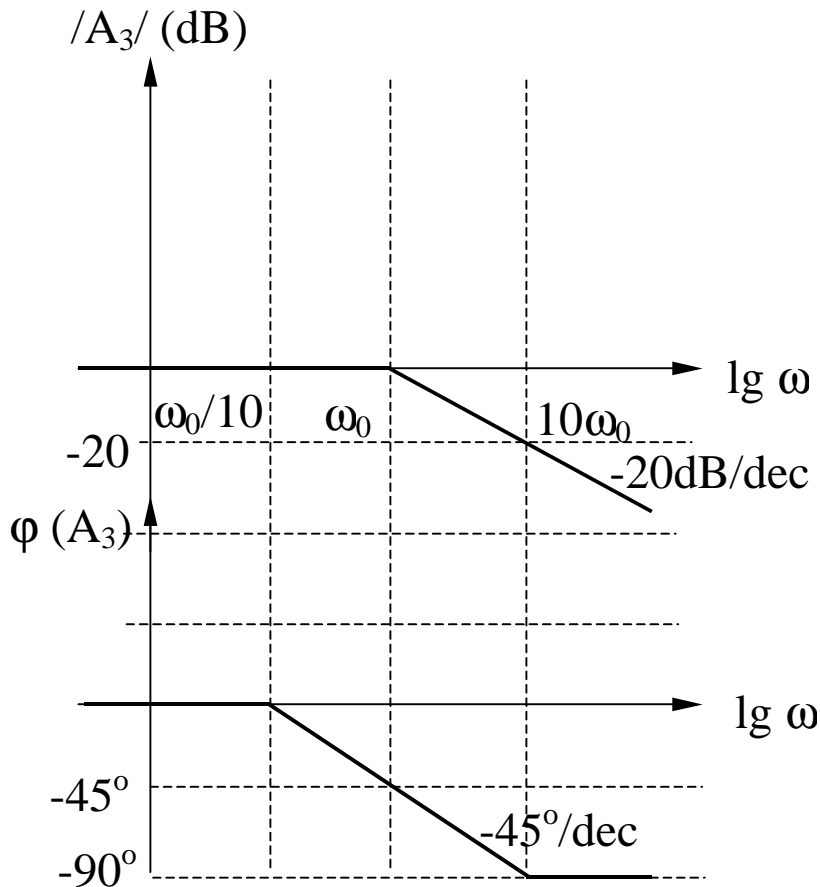
(asimptota inalta frecventa)

$$\omega = \omega_0 \Rightarrow |A_3| = -20 \lg \sqrt{1+1} = -3dB$$

$$\omega = \frac{\omega_0}{2} \Rightarrow |A_3| = -20 \lg \sqrt{1 + \frac{1}{4}} = -1dB$$

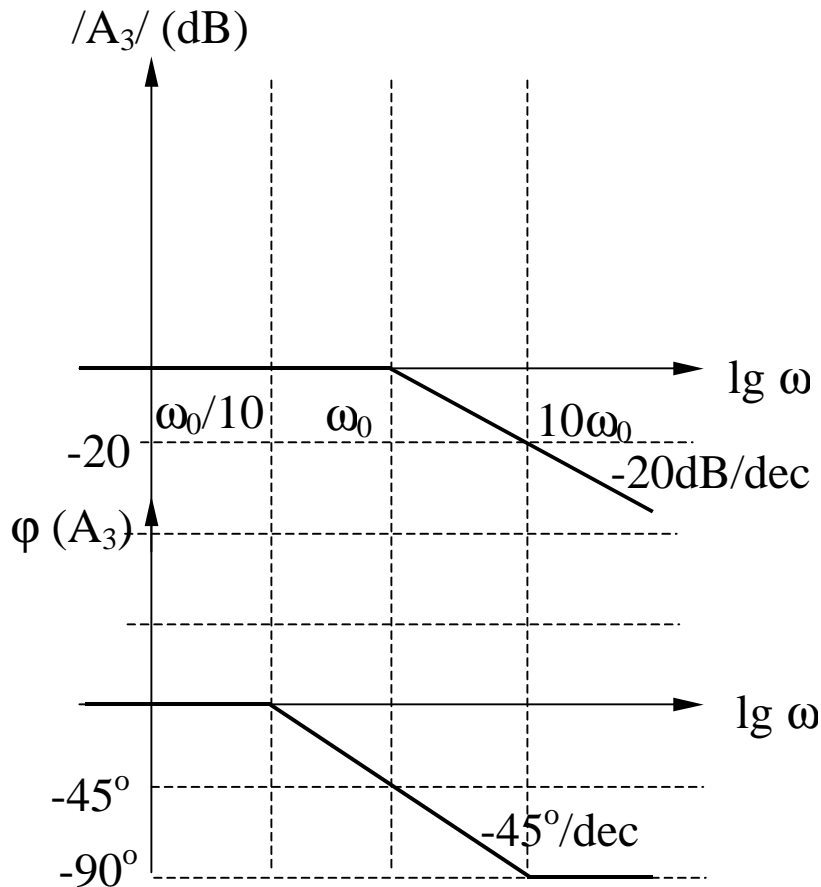
$$\omega = 2\omega_0 \Rightarrow |A_3| = -20 \lg \sqrt{1+4} \cong -7dB$$

$$\Rightarrow \Delta|A_3| = ((-7) - (-6))dB = -1dB$$



## Pol real negativ ( $A_3$ ) - continuare

### Faza



$$\varphi(A_3) = -\arctg\left(\frac{\omega}{\omega_0}\right)$$

$$\omega \ll \omega_0 \Rightarrow \varphi(A_3) = -\arctg(0) = 0^\circ$$

(asimptota joasa frecventa)

$$\omega \gg \omega_0 \Rightarrow \varphi(A_3) = -\arctg(\infty) = -90^\circ$$

(asimptota inalta frecventa)

$$\omega = \omega_0 \Rightarrow \varphi(A_3) = -\arctg(1) = -45^\circ$$

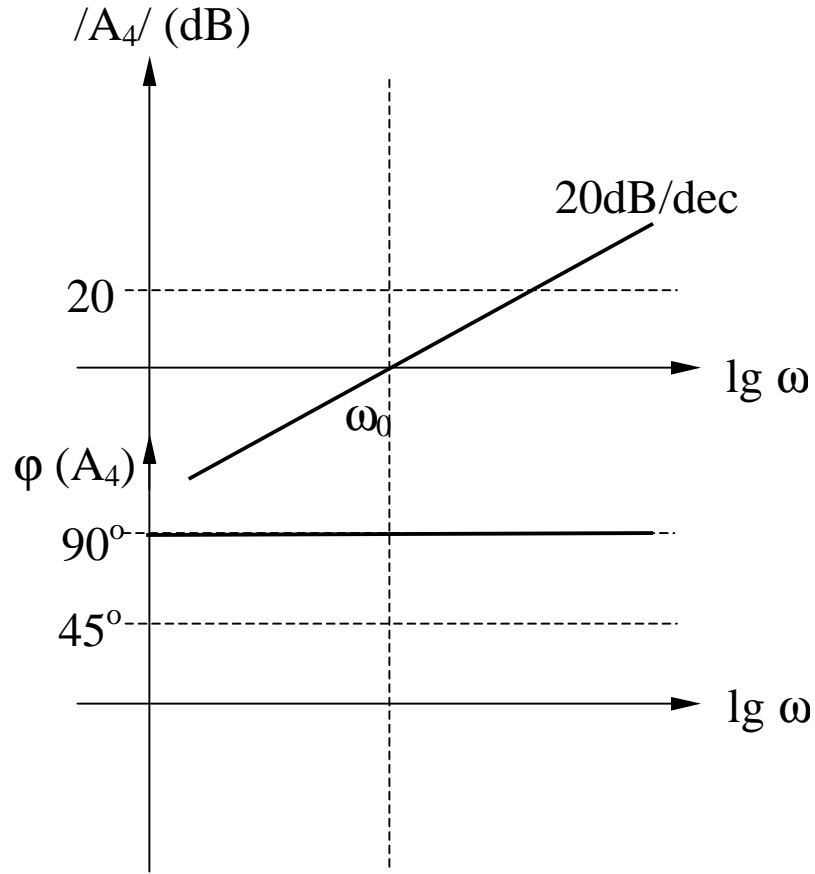
$$\omega = 10\omega_0 \Rightarrow \varphi(A_3) = -\arctg(10) \cong -84^\circ$$

(eroare  $6^\circ$ )

$$\omega = \omega_0 / 10 \Rightarrow \varphi(A_3) = -\arctg(0,1) \cong -6^\circ$$

(eroare  $6^\circ$ )

## Zero simplu in origine ( $A_4$ )

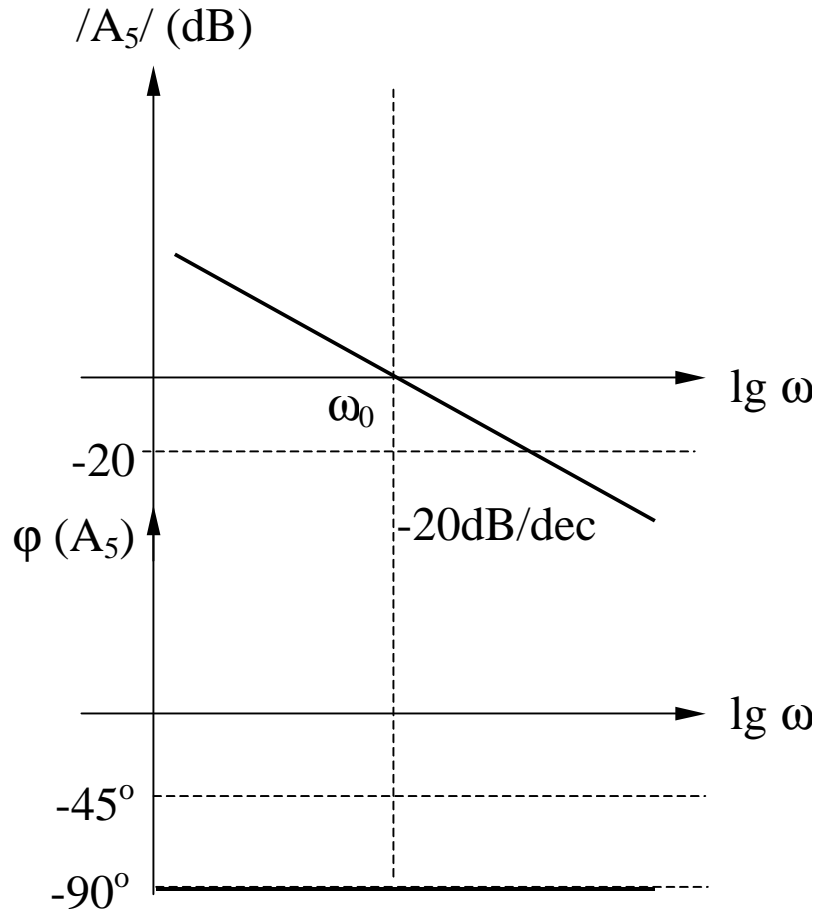


$$A_4 = j \frac{\omega}{\omega_0}$$

$$|A_4| = 20 \lg \left( \frac{\omega}{\omega_0} \right)$$

$$\varphi(A_4) = 90^\circ$$

## Pol simplu in origine ( $A_5$ )

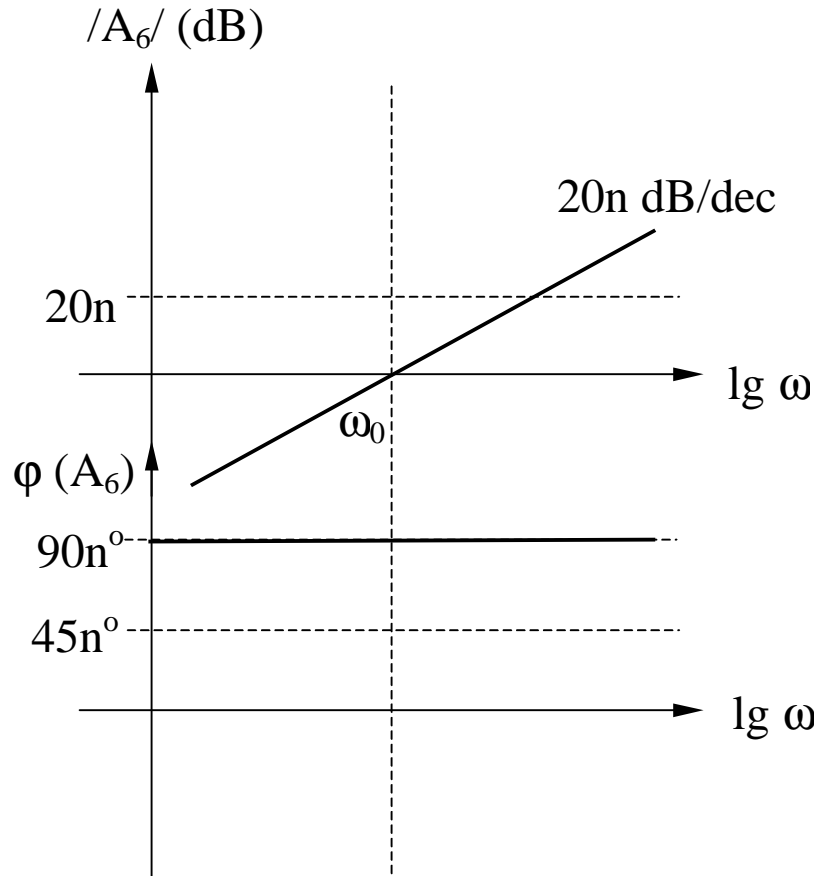


$$A_5 = \frac{1}{j \frac{\omega}{\omega_0}}$$

$$|A_5| = -20 \lg \left( \frac{\omega}{\omega_0} \right)$$

$$\varphi(A_5) = -90^\circ$$

## Zero multiplu in origine ( $A_6$ )

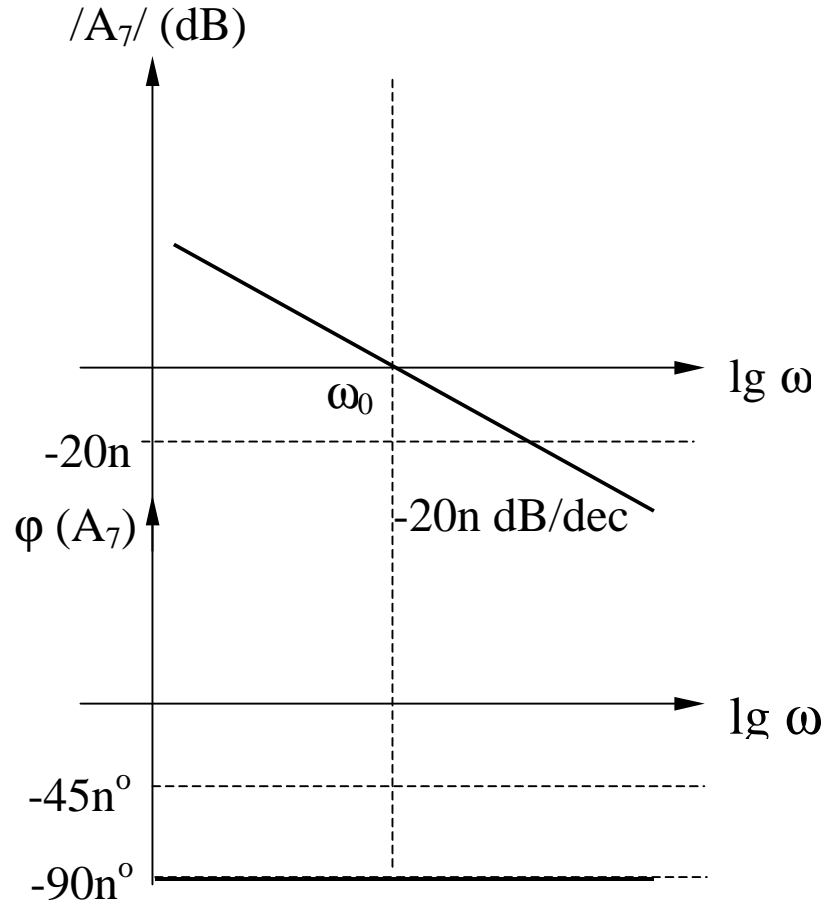


$$A_6 = \left( j \frac{\omega}{\omega_0} \right)^n$$

$$|A_6| = 20 \times n \lg \left( \frac{\omega}{\omega_0} \right)$$

$$\varphi(A_6) = n \times 90^\circ$$

## Pol multiplu in origine ( $A_7$ )

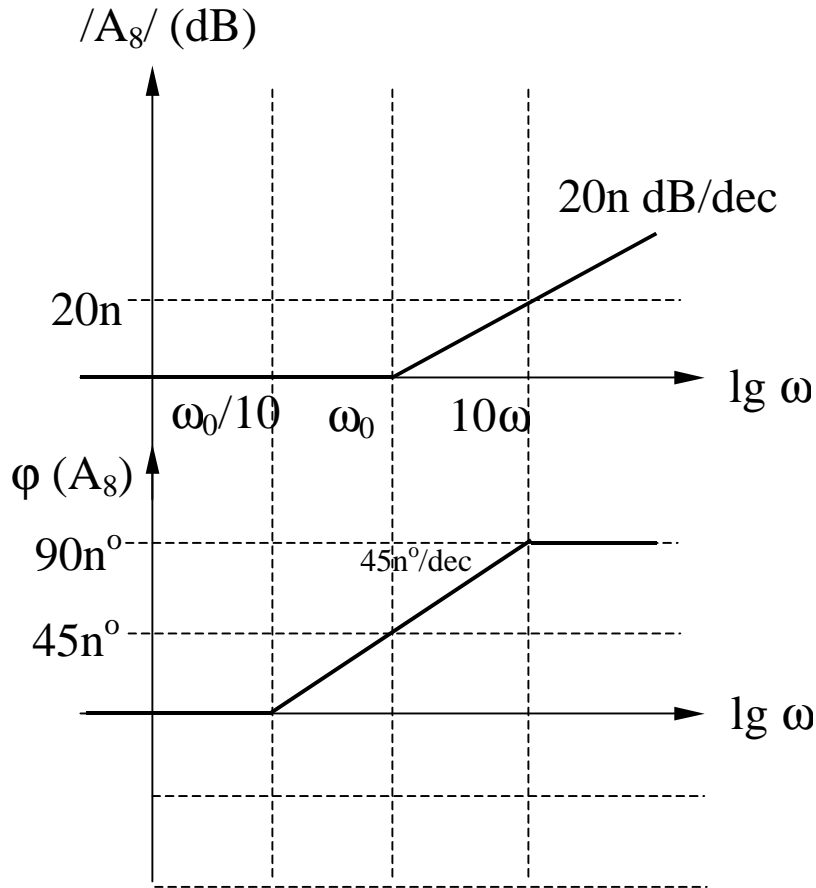


$$A_7 = \frac{1}{\left(j \frac{\omega}{\omega_0}\right)^n}$$

$$|A_7| = -20 \times n \lg \left( \frac{\omega}{\omega_0} \right)$$

$$\varphi(A_7) = -n \times 90^\circ$$

## Zero real negativ multiplu ( $A_8$ )



$$A_8 = \left(1 + j \frac{\omega}{\omega_0}\right)^n$$

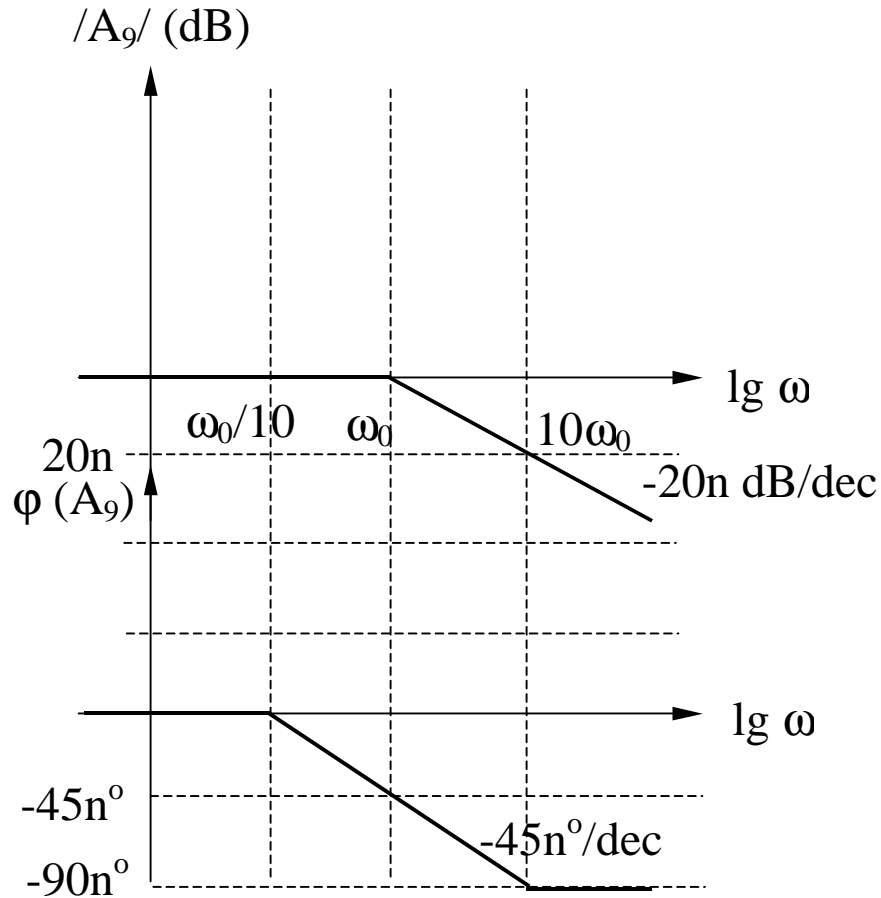
$$|A_8| = 20 \times n \lg \left[ \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2} \right]$$

$$\omega \ll \omega_0 \Rightarrow |A_8| \rightarrow 0$$

$$\omega \gg \omega_0 \Rightarrow |A_8| \rightarrow 20 \times n \lg \left( \frac{\omega}{\omega_0} \right)$$

$$\varphi(A_8) = n \times \arctg \left( \frac{\omega}{\omega_0} \right)$$

## Pol real negativ multiplu ( $A_9$ )



$$A_9 = \frac{1}{\left(1 + j \frac{\omega}{\omega_0}\right)^n}$$

$$|A_9| = -20 \times n \lg \left[ \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2} \right]$$

$$\omega \ll \omega_0 \Rightarrow |A_9| \rightarrow 0$$

$$\omega \gg \omega_0 \Rightarrow |A_9| \rightarrow -20 \times n \lg \left( \frac{\omega}{\omega_0} \right)$$

$$\varphi(A_9) = -n \times \arctg \left( \frac{\omega}{\omega_0} \right)$$



## Factor quadratic ( $A_{10}$ )

$$A_{10}(s) = \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$\omega_0$  = frecventa de rezonanta

$\xi$  = factor de amortizare

Q = factor de calitate

$$\xi Q = \frac{1}{2}$$

Determinarea polilor: din ecuatia caracteristica:

$$s^2 + 2\xi\omega_0 s + \omega_0^2 = s^2 + \frac{\omega_0}{Q}s + \omega_0^2 = 0$$

Rezulta:

$$p_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

## Factor quadratic ( $A_{10}$ ) - continuare

$$p_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

### **Situatii posibile:**

1.  $Q < 0,5$  ( $\xi > 1$ )  $\Rightarrow$  2 poli reali negativi
2.  $Q = 0,5$  ( $\xi = 1$ )  $\Rightarrow$  pol dublu
3.  $Q > 0,5$  ( $\xi < 1$ )  $\Rightarrow$  2 poli complex conjugati
4.  $Q \rightarrow \infty$  ( $\xi \rightarrow 0$ )  $\Rightarrow$  2 poli imaginari

- situatii deja analizate

## Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

$$p_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

$$\operatorname{Re}(p_1, p_2) = -\xi\omega_0 = -\frac{\omega_0}{2Q}$$

$$\operatorname{Im}(p_1, p_2) = \pm\omega_0\sqrt{1-\xi^2} = \pm\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

$$|p_1, p_2| = \omega_0$$

$$\varphi(p_1, p_2) = \frac{\operatorname{Im}(p_1, p_2)}{\operatorname{Re}(p_1, p_2)} = \pm \operatorname{arctg} \sqrt{\frac{1}{\xi^2} - 1} = \pm \operatorname{arctg} \sqrt{4Q^2 - 1}$$

## Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

### Caracteristica de frecventa

$$A_{10}(s) = \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$F(j\omega) = \omega_0^2 A_{10}(j\omega) = \frac{1}{1 + 2j\xi \frac{\omega}{\omega_0} + \left(j \frac{\omega}{\omega_0}\right)^2} = \frac{1}{1 + \frac{j}{Q} \frac{\omega}{\omega_0} + \left(j \frac{\omega}{\omega_0}\right)^2}$$

Se noteaza:  $u = \omega / \omega_0$ ;  $x = (\omega / \omega_0)^2$ . Rezulta:

$$F(ju) = \frac{1}{1 + 2j\xi u - u^2} = \frac{1}{1 + \frac{j}{Q}u - u^2}$$

$$|F(ju)| = \frac{1}{\sqrt{(1-u^2)^2 + 4\xi^2 u^2}}; \quad |F(jx)| = \frac{1}{\sqrt{(1-x)^2 + 4\xi^2 x}}$$

$$\varphi(ju) = -\arctg \frac{2\xi u}{1-u^2}$$

## Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

### Caracteristica de frecventa (continuare)

$$|F(ju)|_{dB} = -20 \lg \sqrt{(1-u^2)^2 + 4\xi^2 u^2}; \quad |F(jx)|_{dB} = -20 \lg \sqrt{(1-x)^2 + 4\xi^2 x}$$

Se noteaza:  $f(x) = (1-x)^2 + 4\xi^2 x = 1 + (4\xi^2 - 2)x + x^2$

Conditia de minim a functiei  $f(x)$  se obtine prin anularea derivatei acesteia:

$$f'(x) = (4\xi^2 - 2) + 2x = 0 \Rightarrow x = 1 - 2\xi^2 > 0 \Rightarrow \xi < \frac{1}{\sqrt{2}} \cong 0,707$$

$$f''(x) = 2 > 0$$

**Deci**, functia  $f(x)$  va avea un minim pentru  $x = 1 - 2\xi^2$  (echivalent cu **existenta unui maxim al  $|F(jx)|_{dB}$** ) **numai daca  $\xi < 0,707$  (sau  $Q > 0,707$ ).**

Se va obtine un maxim pentru  $\omega_p$  avand expresia:

$$x = 1 - 2\xi^2 \Leftrightarrow \omega_{Peak} = \omega_P = \omega_0 \sqrt{1 - 2\xi^2} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

## Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

### Caracteristica de frecventa (continuare)

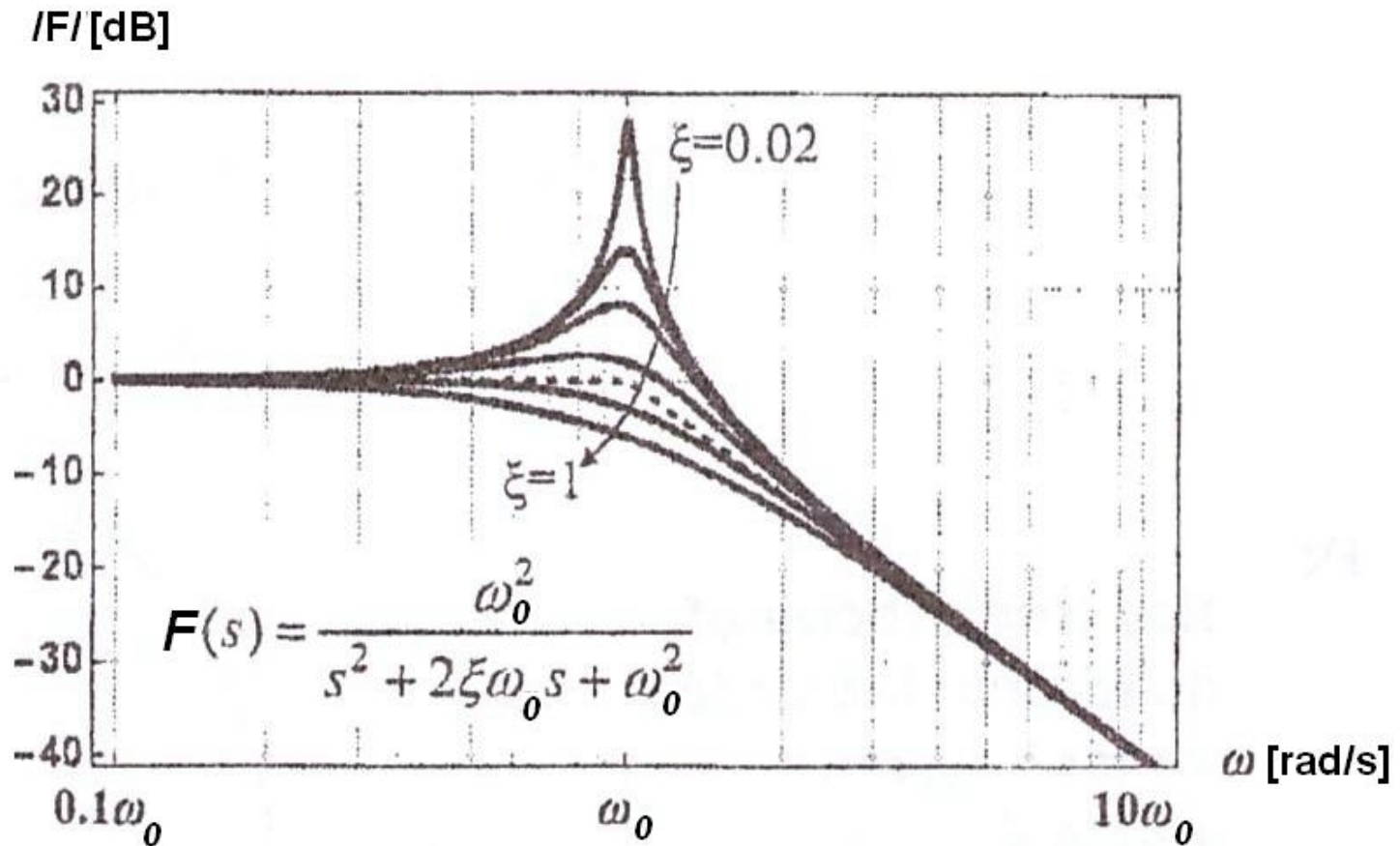
Valoarea acestui maxim este:

$$|F(jx)|_{dB} = -20 \lg \sqrt{(1 - 1 + 2\xi^2)^2 + 4\xi^2(1 - 2\xi^2)}$$

$$|F(jx)|_{dB} = 20 \lg \frac{1}{2\xi\sqrt{1-\xi^2}} = 20 \lg \frac{Q}{\sqrt{1-\frac{1}{4Q^2}}}$$

# Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

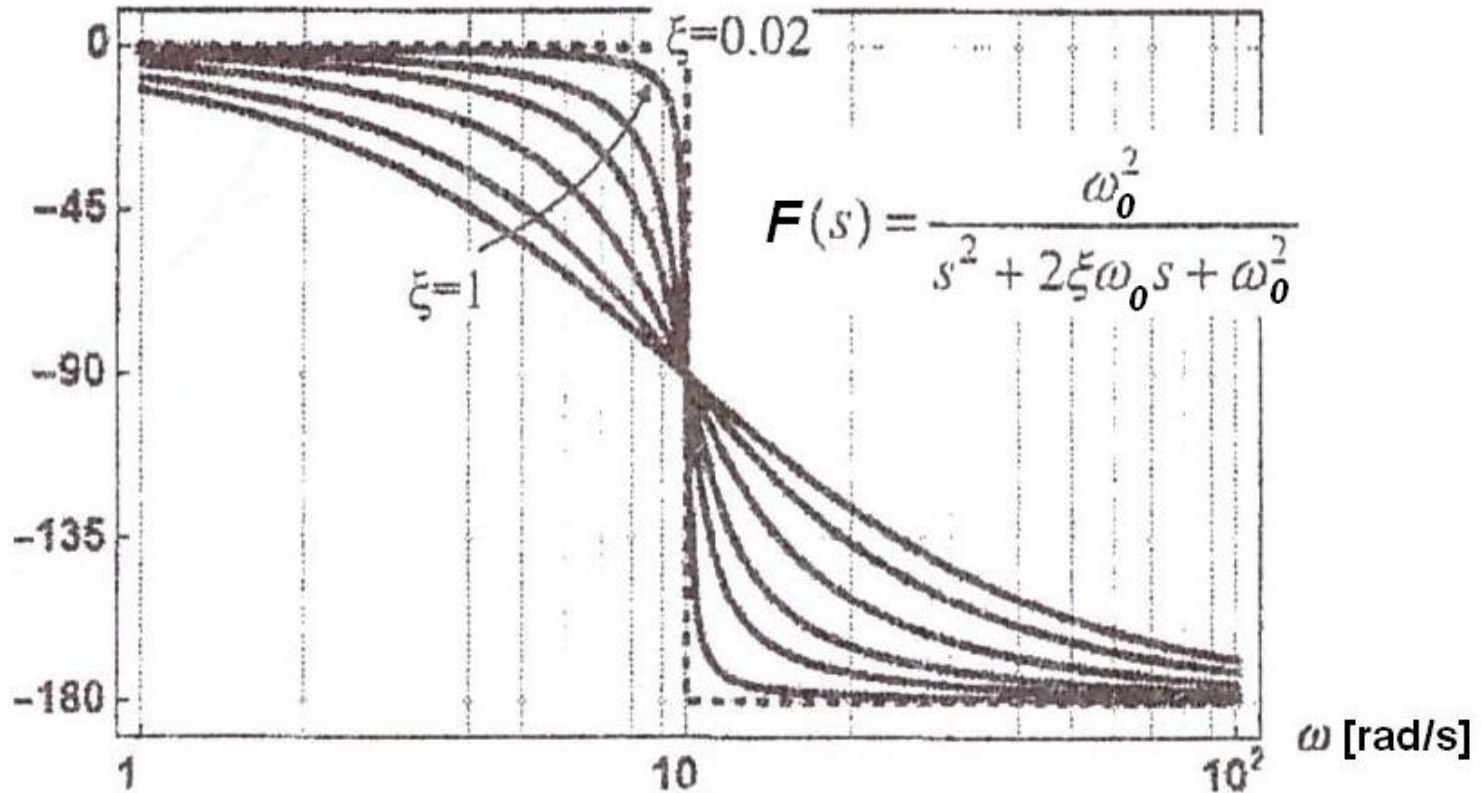
Caracteristica de frecventa (continua)



# Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

## Caracteristica de frecventa (continua)

faza [grade]





## Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

Trasarea diagramelor Bode (MODUL) pentru poli complex conjugati

$$|F(ju)|_{dB} = -20 \lg \sqrt{(1-u^2)^2 + 4\xi^2 u^2}$$

$$|F(j\omega)|_{dB} = -20 \lg \sqrt{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + 4\xi^2 \left(\frac{\omega}{\omega_0}\right)^2}$$

$$\omega \ll \omega_0 \Rightarrow |F(j\omega)|_{dB} \rightarrow 0dB \quad (\text{asimptota joasa frecventa})$$

$$\omega \gg \omega_0 \Rightarrow |F(j\omega)|_{dB} \cong -40 \lg \left(\frac{\omega}{\omega_0}\right) \quad (\text{asimptota inalta frecventa})$$

$$\omega = \omega_0 \Rightarrow |F(j\omega)|_{dB} = -20 \lg(2\xi)$$

## Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

**Trasarea diagramelor Bode (MODUL) pentru polii complex conjugati (continuare)  
(daca EXISTA maxim:  $0 < \xi < 0,707$  sau  $Q > 0,707$ )**

1. Se traseaza asimptotele:

- la JF: 0dB pana in  $\omega_0$

- la IF: o dreapta cu panta de -40dB/decada pornind din  $\omega_0$

2. Se calculeaza  $\omega = \omega_p$  pentru care se obtine maximul:

$$\omega_p = \omega_0 \sqrt{1 - 2\xi^2} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

3. Se calculeaza valoarea maximului:

$$|F_p|_{dB} = 20 \lg \frac{1}{2\xi \sqrt{1 - \xi^2}} = 20 \lg \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

4. Se traseaza o curba care tinde asimptotic catre cele 2 axe (JF si IF) si care trece prin punctul de maxim ( $\omega_p, |F_p|_{dB}$ )

## **Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )**

**Trasarea diagramelor Bode (MODUL) pentru polii complex conjugati (continuare)  
(daca NU EXISTA maxim:  $\xi > 0,707$  sau  $Q < 0,707$ )**

1. Se traseaza asimptotele:

- la JF: 0dB pana in  $\omega_0$

- la IF: o dreapta cu panta de -40dB/decada pornind din  $\omega_0$

2. Se traseaza o curba care tinde asimptotic catre cele 2 axe (JF si IF)

## Factor quadratic ( $A_{10}$ ) – poli complex conjugati ( $Q > 0,5$ )

Trasarea diagramelor Bode (FAZA) pentru polii complex conjugati

$$\varphi(ju) = -\arctg \frac{2\xi u}{1-u^2} = -\arctg \frac{\frac{1}{Q}u}{1-u^2}$$
$$\varphi(j\omega) = -\arctg \frac{2\xi \frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2} = -\arctg \frac{\frac{1}{Q} \frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}$$

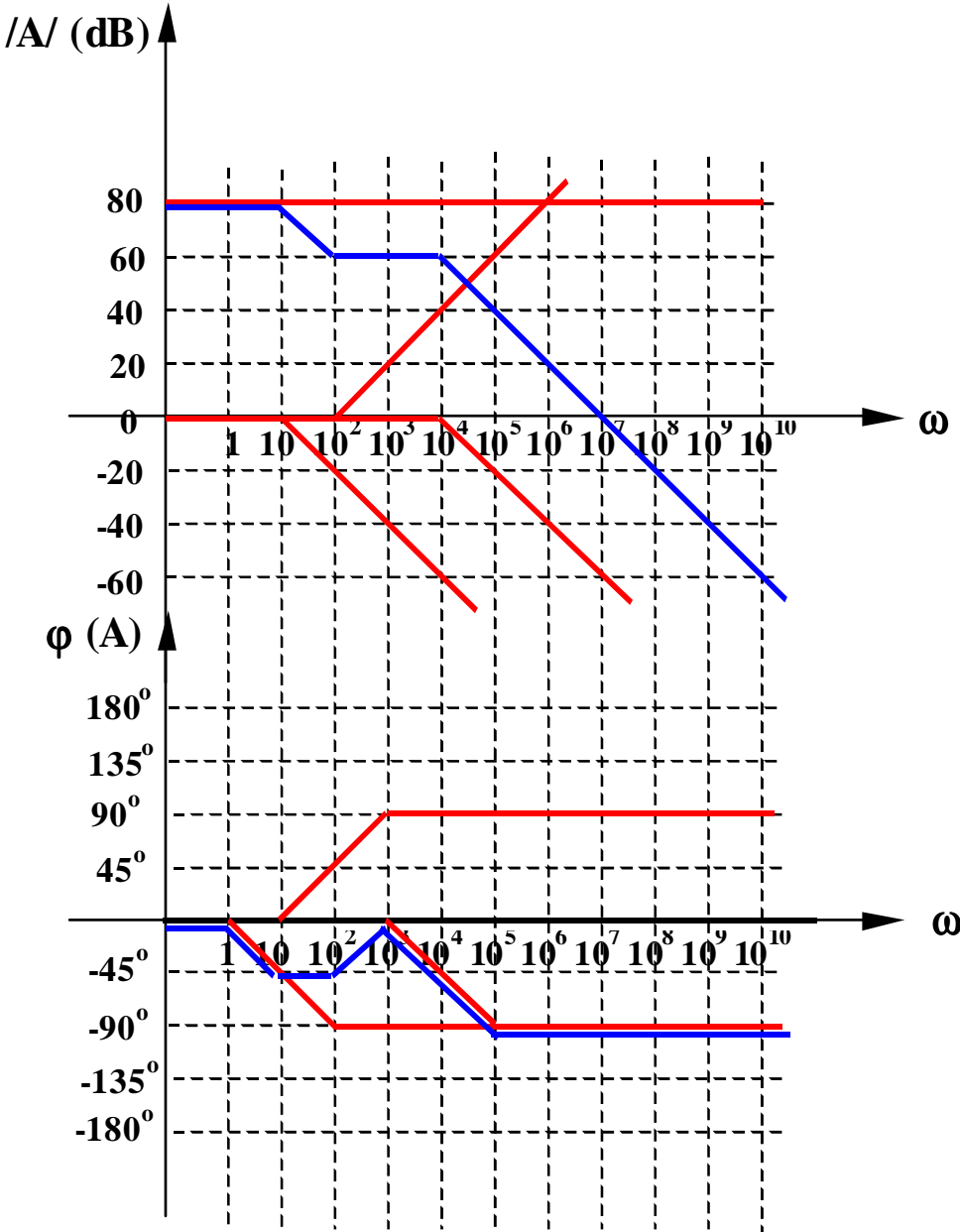
$$\omega \ll \omega_0 \Rightarrow \varphi \rightarrow -\arctg 0 = 0 \quad (\text{asimptota joasa frecventa})$$

$$\omega \gg \omega_0 \Rightarrow \varphi \rightarrow -\arctg \infty = -180^\circ \quad (\text{asimptota inalta frecventa})$$

$$\omega = \omega_0 \Rightarrow \varphi = \arctg(\infty) = -90^\circ$$

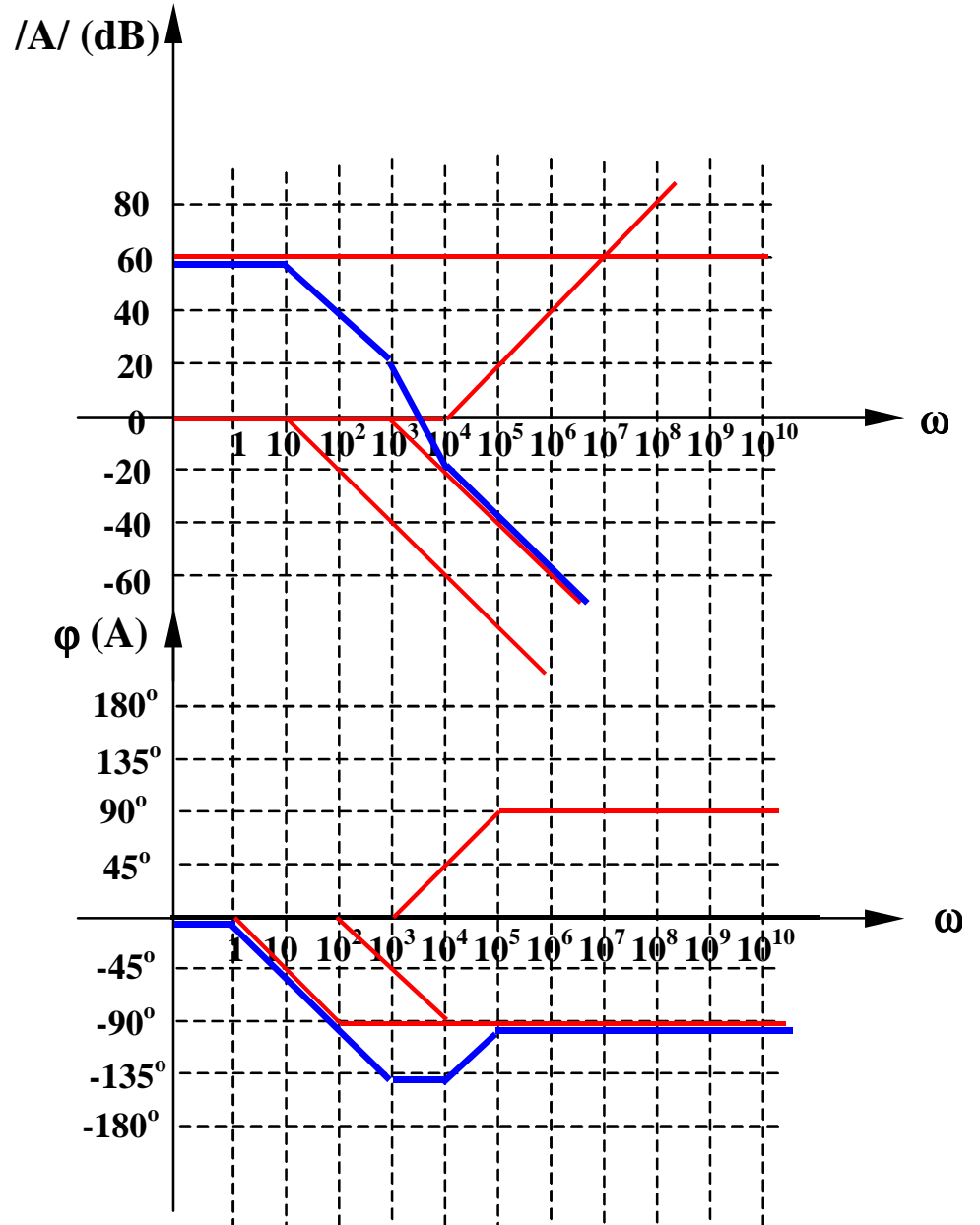
# Exemplul 1

$$A(j\omega) = 10^4 \frac{1 + j \frac{\omega}{10^2}}{\left(1 + j \frac{\omega}{10}\right) \left(1 + j \frac{\omega}{10^4}\right)}$$



# Exemplul 2

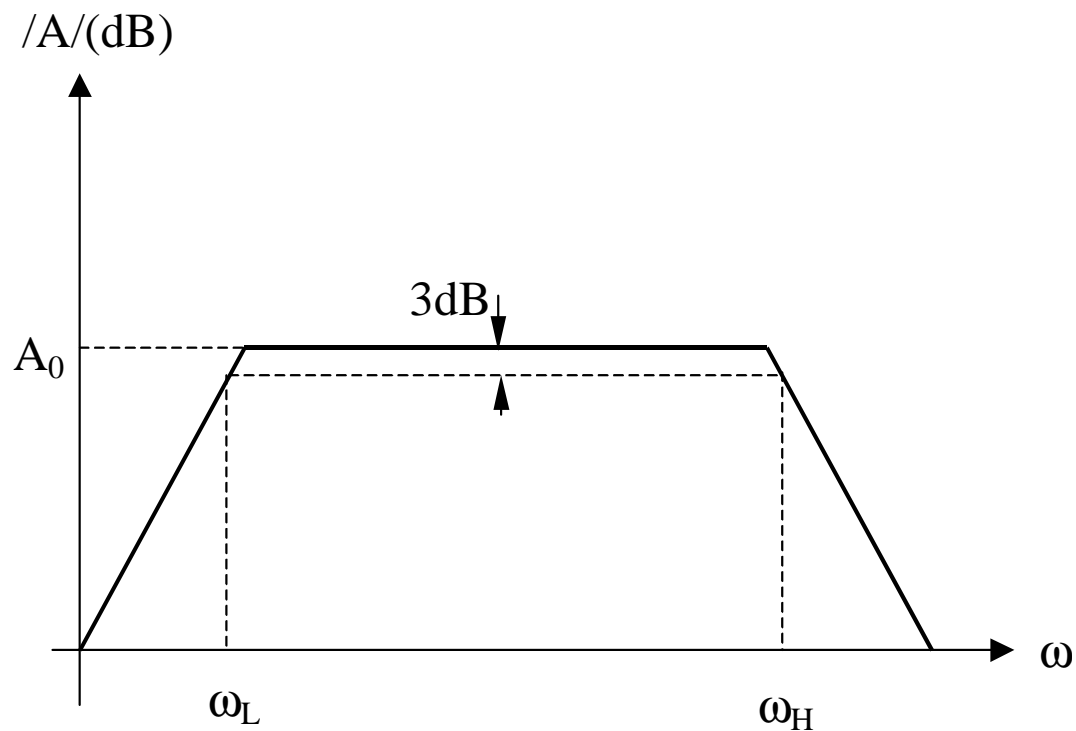
$$A(j\omega) = 10^3 \frac{1 + j \frac{\omega}{10^4}}{\left(1 + j \frac{\omega}{10}\right) \left(1 + j \frac{\omega}{10^3}\right)}$$



## **7.2. Raspunsul in frecventa al amplificatoarelor**

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.1. Banda de frecventa





## **7.2. Raspunsul in frecventa al amplificatoarelor**

### **7.2.1. Banda de frecventa**

#### **La frecvente medii:**

- condensatoarele de cuplaj si de decuplare sunt scurt-circuitate
- condensatoarele interne ale dispozitivelor sunt circuitate deschise

#### **La frecvente joase:**

- condensatoarele de cuplaj si de decuplare nu mai sunt scurt-circuitate
- condensatoarele interne ale dispozitivelor sunt circuitate deschise

#### **La frecvente inalte:**

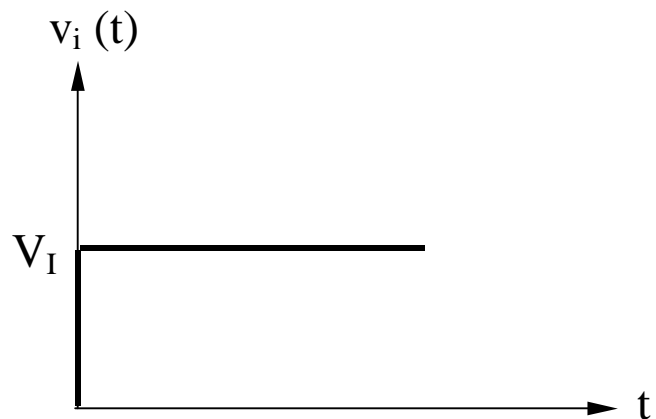
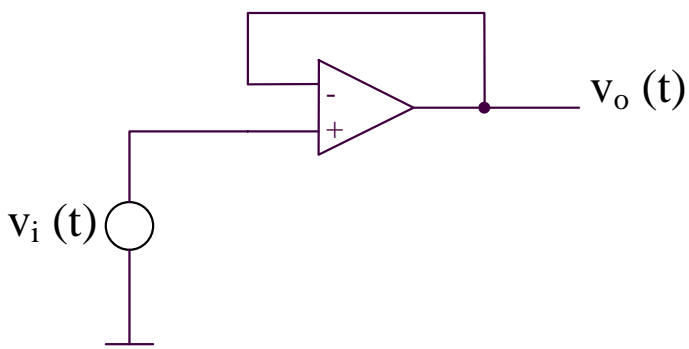
- condensatoarele de cuplaj si de decuplare sunt scurt-circuitate
- condensatoarele interne ale dispozitivelor nu mai sunt circuitate deschise

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Slew-Rate-ul (SR) reprezinta viteza maxima de crestere a tensiunii de iesire a unui amplificator operational in CONDITII DE SEMNAL MARE.

#### Evaluarea performantei la semnal mare si inalta frecventa pentru un AO



Amplificarea in bucla inchisa are expresia:

$$A = \frac{a}{1 + af}$$

Se considera un AO cu un singur pol:

$$a = \frac{a_0}{1 + \frac{s}{\omega_H}}$$

Circuitul fiind repetor,  $f = 1$ . Se obtine:

$$A(s) = \frac{V_o(s)}{V_i(s)} = \frac{a(s)}{1 + a(s)} = \frac{1}{1 + \frac{1}{a(s)}} \cong \frac{1}{1 + \frac{s}{\omega_u}}$$

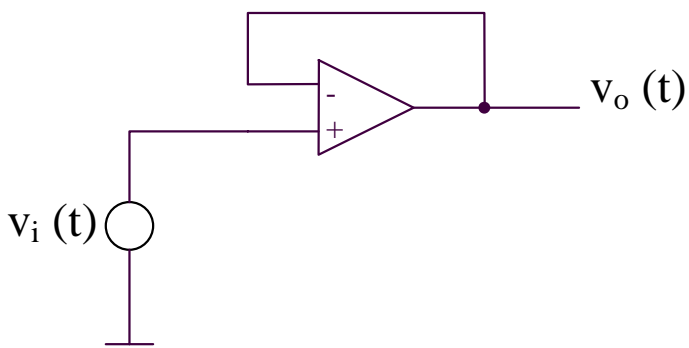
unde:

$$\omega_u = a_0 \omega_H$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Evaluarea performantei la semnal mare si inalta frecventa pentru un AO



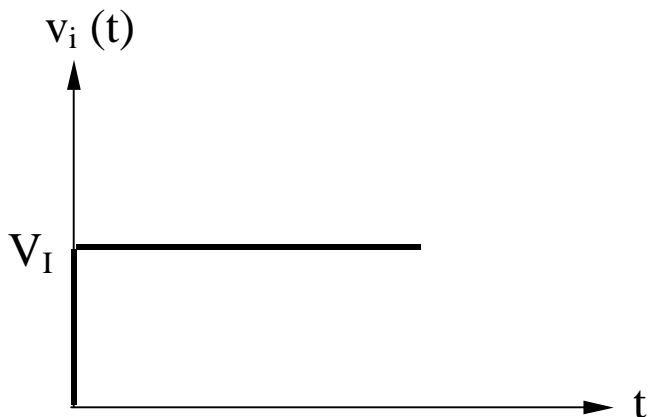
Semnalul de intrare are transformata Laplace:

$$V_i(s) = \frac{V_I}{s}$$

Rezulta:

$$V_o(s) = V_i(s)A(s) = \frac{V_I}{s} \frac{1}{1 + \frac{s}{\omega_u}} =$$

$$= V_I \left( \frac{1}{s} - \frac{1}{s + \omega_u} \right)$$



Deci:

$$v_o(t) = V_I (1 - e^{-\omega_u t})$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Evaluarea performantei la semnal mare si inalta frecventa pentru un AO

$$V_I \left(1 - e^{-\omega_u t_{90\%}}\right) = 0,9V_I \Rightarrow t_{90\%} = -\frac{\ln(0,1)}{\omega_u}$$

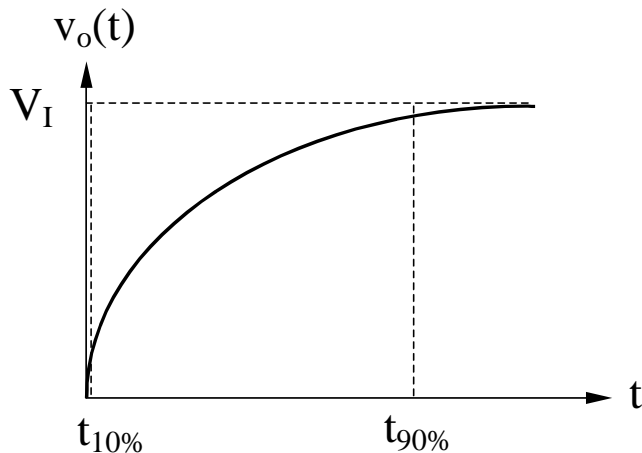
$$V_I \left(1 - e^{-\omega_u t_{10\%}}\right) = 0,1V_I \Rightarrow t_{10\%} = -\frac{\ln(0,9)}{\omega_u}$$

$$t_r = t_{90\%} - t_{10\%} = \frac{\ln(9)}{\omega_u} \cong \frac{2,2}{\omega_u}$$

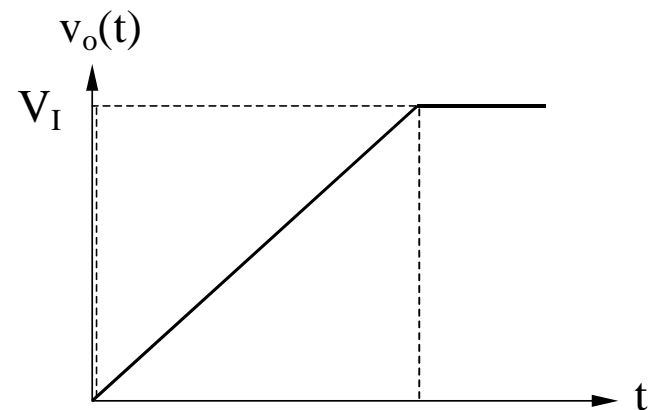
Rezulta:

$$t_r \omega_u = 2,2 \quad \text{sau} \quad t_r f_u = 0,35$$

Raspuns in timp estimat



Raspuns in timp masurat



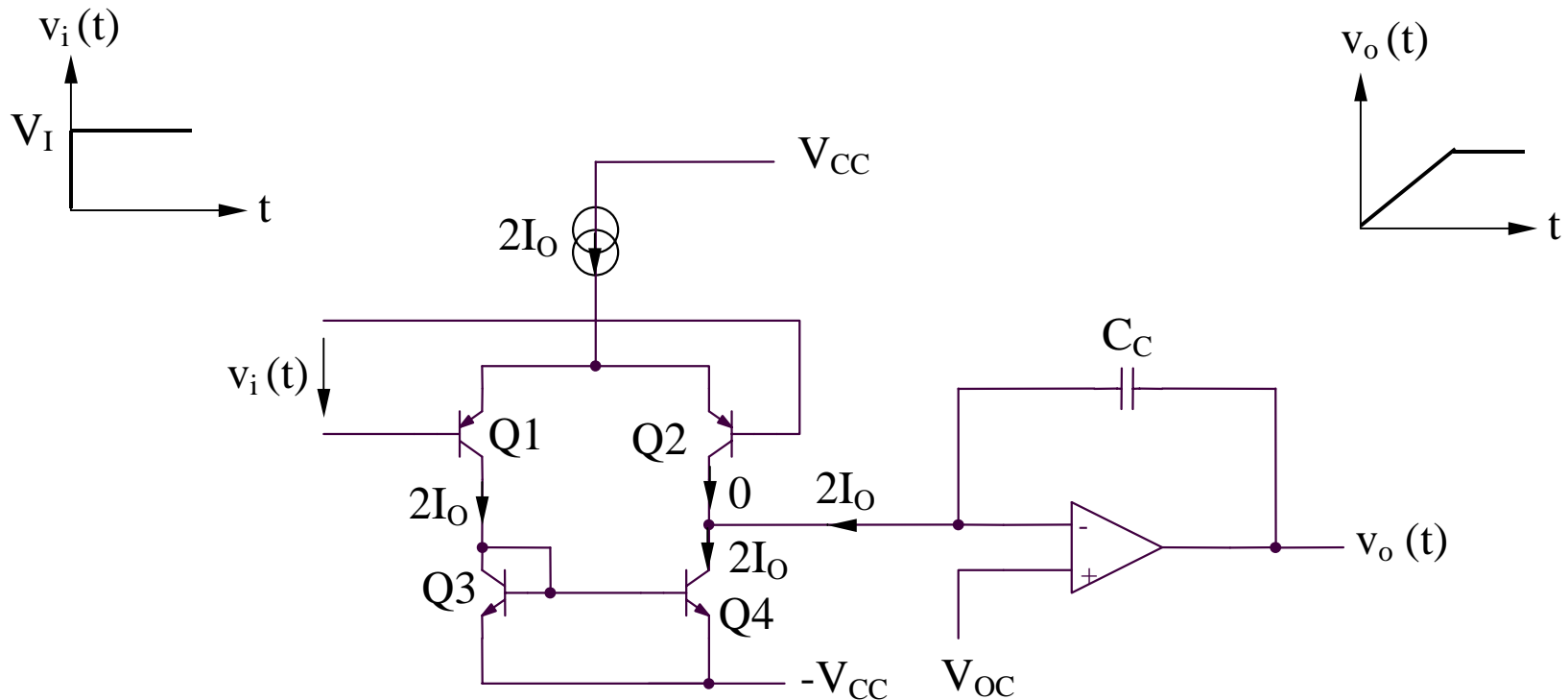
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

**Evaluarea performantei la semnal mare si inalta frecventa pentru un AO**

Diferenta majora intre cele doua raspunsuri in timp este cauzata de faptul ca analiza de SEMNAL MIC nu poate fi utilizata pentru a determina comportamentul circuitului in conditii de SEMNAL MARE.

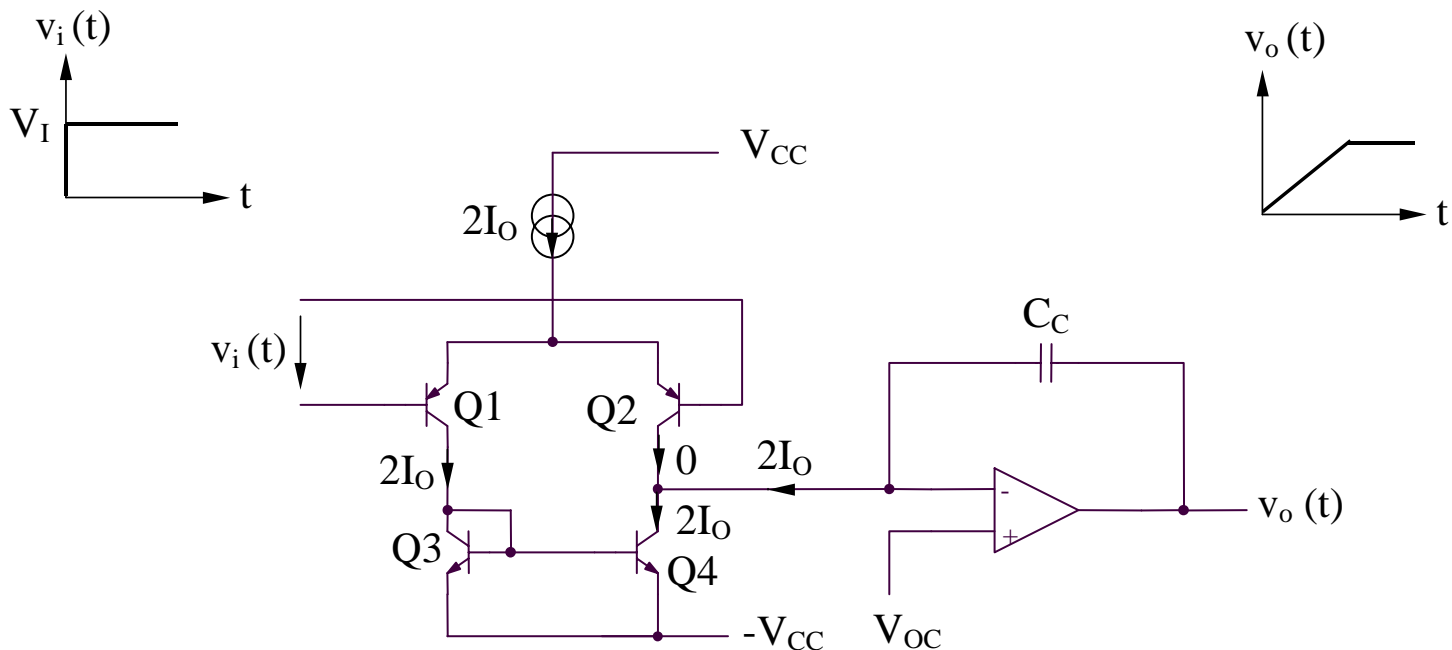
Structura tipica a unui AO este:



## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

Evaluarea performantei la semnal mare si inalta frecventa pentru un AO



La  $t = 0$ , semnalul de intrare creste de la 0 la  $V_I$  (de ordinul voltilor), dar tensiunea de iesire nu raspunde instantaneu. Tensiunea mare de intrare aplicata AO va scoate complet AD de intrare din zona liniara. Prin urmare,  $i_{C2} = 0$ , iar  $i_{C1} = i_{C3} = i_{C4} = 2I_O$ , curent ce va incarca condensatorul  $C_C$  la o tensiune  $v_O$ , cu o panta  $dv_O/dt = SR$ :

$$v_O = v_{C_C} = \frac{1}{C_C} \int_0^t 2I_O dt \quad \frac{dv_O}{dt} = \frac{2I_O}{C_C} = SR = \text{constant}$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.2. Slew-Rate-ul (SR) amplificatoarelor operationale

**Efectul limitarilor introduse de SR asupra functionarii la semnal mare de intrare de tip sinusoidal**

Ideal, tensiunea de iesire va urmari tensiunea de intrare:

$$v_o = V_o \sin \omega t$$

cu o viteza de variatie maxima a tensiunii de iesire avand expresia:

$$\left. \frac{dv_o}{dt} \right|_{max} = \omega V_o$$

a. Daca  $\left. \frac{dv_o}{dt} \right|_{max} < SR$ ,  $v_o$  va urmari perfect tensiunea de intrare

b. Daca  $\left. \frac{dv_o}{dt} \right|_{max} > SR$ ,  $v_o$  va fi afectata de distorsiuni puternice

Frecventa maxima a semnalului de iesire de amplitudine maxima (aproximativ egala cu tensiunea de alimentare) nedistorsionat,  $f_{max}$ , se poate determina astfel:

$$\left. \frac{dv_o}{dt} \right|_{max} = \omega_{max} V_{OM} = SR \Rightarrow f_{max} = \frac{SR}{2\pi V_{OM}}$$

## **7.2. Raspunsul in frecventa al amplificatoarelor**

### **7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare**

#### **A. Analiza directa**

- presupune: - realizarea circuitelor echivalente pornind de la modelele de inalta frecventa ale dispozitivelor constitutive
  - determinarea setului de ecuatii specifice circuitului analizat
  - rezolvarea acestor ecuatii
- prezinta avantajul unui raspuns in frecventa exact
- prezinta dezavantajul major al unei complexitati de calcul foarte ridicate
- este utilizata in special in simulatoare

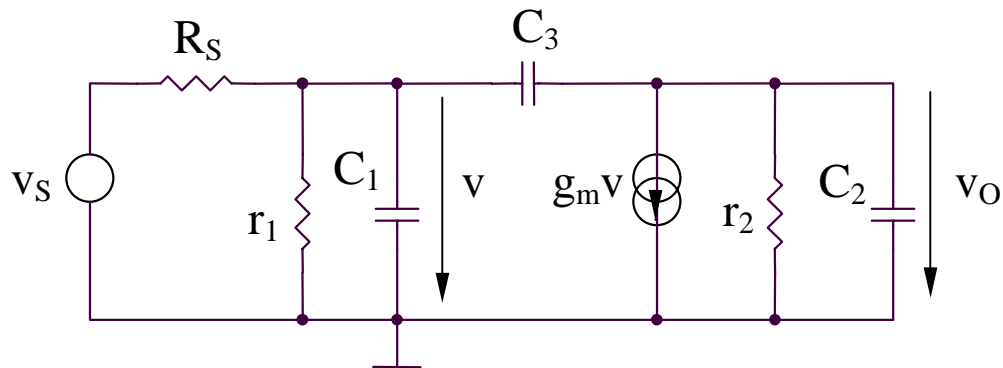


## 7.2. Raspunsul in frecventa al amplificatoarelor

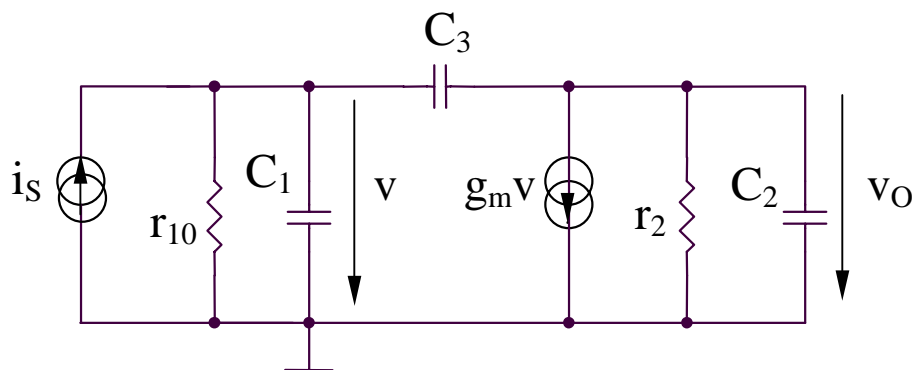
### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

**Exemple: etajele emitor comun/sursa comuna**

Modelul general al etajelor emitor comun/sursa comuna:



Circuitul echivalent (echivalare Norton):



$$i_S = \frac{v_S}{R_S}$$

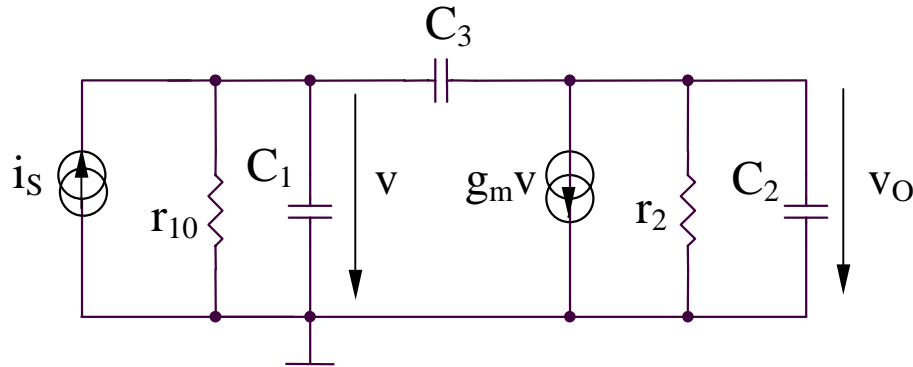
$$r_{10} = R_S // r_1$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

Exemple: etajele emitor comun/sursa comuna

Circuitul echivalent (echivalare Norton):



Ecuatiile potentialelor la noduri:

$$v(sC_1 + sC_3 + g_{10}) = i_S + v_O sC_3$$

$$v_O(sC_2 + sC_3 + g_2) = -g_m v + v sC_3$$

Rezulta:

$$\frac{v_O}{v_S} = -g_m r_2 \frac{r_1}{r_1 + R_S} \frac{1 - s \frac{C_3}{g_m}}{1 + s \left[ \frac{C_2 + C_3}{g_2} + \frac{C_1 + C_3}{g_1 + G_S} + \frac{g_m C_3}{g_2 (g_1 + G_S)} \right] + s^2 \frac{C_1 C_2 + C_1 C_3 + C_2 C_3}{(g_1 + G_S) g_2}}$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

**Exemple: etajele emitor comun/sursa comuna**

Considerand ca circuitul are doi poli reali,  $p_1$  si  $p_2$ , numitorul  $P(s)$  al amplificarii poate fi scris astfel:

$$P(s) = \left(1 - \frac{s}{p_1}\right) \left(1 - \frac{s}{p_2}\right) = 1 - \left(\frac{1}{p_1} + \frac{1}{p_2}\right)s + \frac{1}{p_1 p_2} s^2$$

$$P(s) = 1 + a_1 s + a_2 s^2$$

unde:

$$a_1 = -\left(\frac{1}{p_1} + \frac{1}{p_2}\right) \qquad a_2 = \frac{1}{p_1 p_2}$$

Daca se presupune ca exista un pol dominant de inalta frecventa,  $|p_1| \ll |p_2|$ , rezulta:

$$p_1 = -\frac{1}{a_1} \qquad p_2 = -\frac{a_1}{a_2}$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

**Exemple: etajele emitor comun/sursa comuna**

Se obtin urmatoarele expresii aproximative ale polilor:

$$p_1 = -\frac{1}{a_1} = -\frac{1}{\frac{C_2 + C_3}{g_2} + \frac{C_1 + C_3}{g_1 + G_S} + \frac{g_m C_3}{g_2(g_1 + G_S)}} = -\frac{1}{(r_1 // R_S)(C_1 + C_3) + r_2(C_2 + C_3) + g_m r_2(r_1 // R_S)C_3}$$

$$p_2 = -\frac{a_1}{a_2} = -\frac{(g_1 + G_S)(C_2 + C_3) + g_2(C_1 + C_3) + g_m C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3}$$

$$p_2 \cong -\frac{g_m C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3} = -\frac{g_m}{C_1 + C_2 + \frac{C_1 C_2}{C_3}}$$

**Concluzii:**

- analiza directa necesita un efort de calcul considerabil
- pentru analiza functionarii circuitului la frecvente inalte, daca zeroul nu este dominant, polul cel mai important este acela de modul minim, el determinand valoarea  $\omega_H$  ( $\omega_H = 1/p_1$ )
- daca  $C_3$  creste,  $\omega_{p1}$  scade si  $\omega_{p2}$  creste, producand indepartarea polilor (poles splitting) si justificand presupunerea  $1/p_1 \ll 1/p_2$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

**Exemple: etajele emitor comun/sursa comuna**

**Cazuri particulare**

**Sursa de tensiune de intrare ideala ( $R_S = 0$ ,  $G_S = \infty$ )**

$$p_1 = -\frac{g_2}{C_2 + C_3} \qquad p_2 = -\infty$$

Amplificatorul are un singur pol.

**Sursa de curent de intrare ideala ( $R_S = \infty$ ,  $G_S = 0$ ). Considerand  $g_m \gg g_1, g_2$ , rezulta:**

$$p_1 = -\frac{g_1 g_2}{g_m C_3} \qquad p_2 \cong -\frac{g_m}{C_1 + C_2 + \frac{C_1 C_2}{C_3}}$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### **B. Metoda constantelor de timp**

- se poate utiliza in situatia existentei unor poli dominanti si a unor zerouri ne-dominante
- permite estimarea aproximativa a  $\omega_L$  si  $\omega_H$

Funcția de transfer a amplificatorului poate fi exprima astfel:

$$A(s) = \frac{a_0 + a_1s + \dots + a_ms^m}{b_0 + b_1s + \dots + b_ns^n} = K \frac{\left(1 - \frac{s}{z_1}\right)\left(1 - \frac{s}{z_2}\right)\dots\left(1 - \frac{s}{z_m}\right)}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\dots\left(1 - \frac{s}{p_n}\right)}, \quad m < n$$

Un caz frecvent este cel in care funcția de transfer are numai poli (sau zerourile nu sunt importante):

$$A(s) = \frac{K}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\dots\left(1 - \frac{s}{p_n}\right)}$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### **B. Metoda constantelor de timp**

Un caz practic important este cel in care unul din poli este dominant:

$$|p_1| \ll |p_2|, \dots, |p_n|$$

Deci:

$$\frac{1}{|p_1|} \gg \left| \sum_{i=2}^n \left( -\frac{1}{p_i} \right) \right|$$

rezultand:

$$|A(j\omega)| = \frac{K}{\sqrt{\left[1 + \left(\frac{\omega}{p_1}\right)^2\right] \left[1 + \left(\frac{\omega}{p_2}\right)^2\right] \dots \left[1 + \left(\frac{\omega}{p_n}\right)^2\right]}} \cong \frac{K}{\sqrt{\left[1 + \left(\frac{\omega}{p_1}\right)^2\right]}}$$

Aproximatia este corecta cel puțin pana la  $\omega = |p_1|$ . Deci, expresia anterioara aproximativa va permite determinarea corecta a  $\omega_{-3dB}$ :

$$\omega_{-3dB} = |p_1|$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### B. Metoda constantelor de timp

##### B1. Estimarea $\omega_L$ utilizand metoda constantelor de timp de scurt-circuit

Pentru un circuit avand n condensatoare de cuplaj si de decuplare,  $\omega_L$  poate fi determinat astfel:

$$\omega_L \cong \sum_{i=1}^n \frac{1}{R_{iS} C_i}$$

$R_{iS}$  reprezinta rezistenta echivalenta dintre terminalele condensatorului  $C_i$ , considerand toate celelalte condensatoare scurt-circuit.



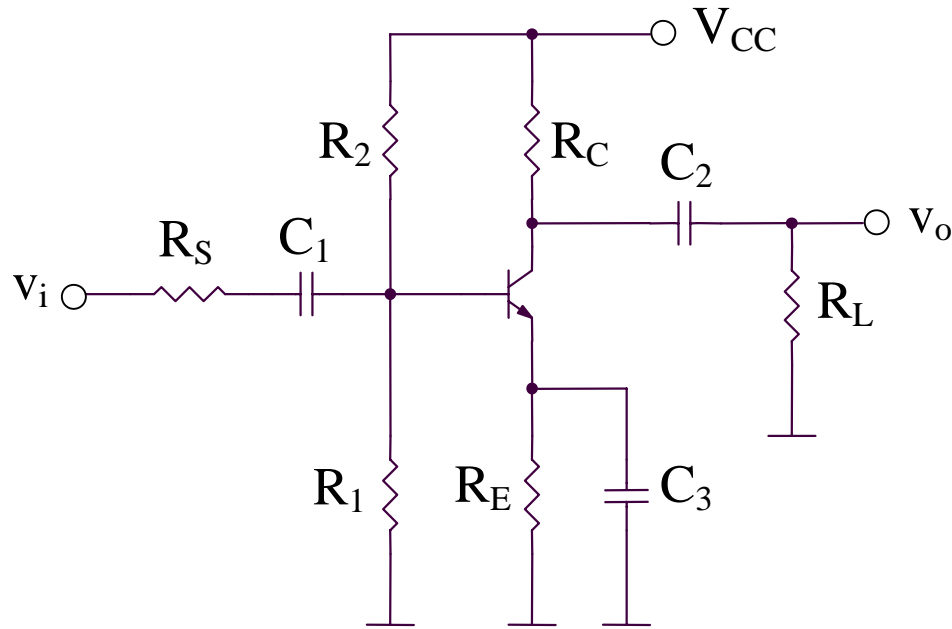
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### B. Metoda constantelor de timp

#### B1. Estimarea $\omega_L$ utilizand metoda constantelor de timp de scurt-circuit

Exemplu: amplificatorul emitor comun



- 3 condensatoare de cuplaj si de decuplare ( $C_1$ ,  $C_2$  si  $C_3$ )
- 2 condensatoare interne ale tranzistorului ( $C_\pi$  si  $C_\mu$ )

## 7.2. Raspunsul in frecventa al amplificatoarelor

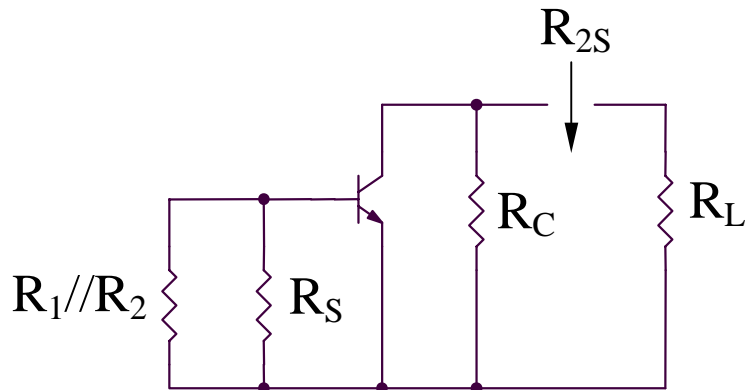
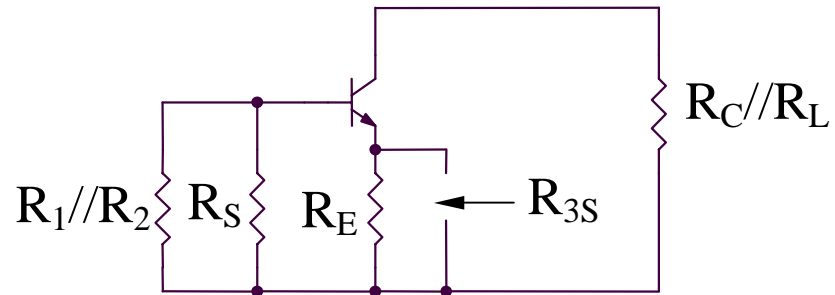
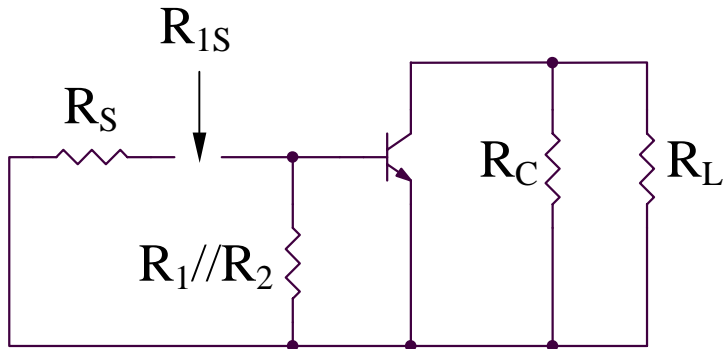
### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### B. Metoda constantelor de timp

#### B1. Estimarea $\omega_L$ utilizand metoda constantelor de timp de scurt-circuit

Exemplu: amplificatorul emitor comun

(continuare)



$$R_{1S} = R_S + R_1 // R_2 // r_{\pi}$$

$$R_{2S} = R_L + R_C // r_o$$

$$R_{3S} = R_E // \frac{r_{\pi} + R_S // R_1 // R_2}{\beta + 1}$$

$$\omega_L \cong \sum_{i=1}^3 \frac{1}{R_{iS} C_i} = \frac{1}{R_{1S} C_1} + \frac{1}{R_{2S} C_2} + \frac{1}{R_{3S} C_3}$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### B. Metoda constantelor de timp

##### B2. Estimarea $\omega_H$ utilizand metoda constantelor de timp de gol

Pentru un circuit avand  $m$  condensatoare,  $\omega_H$  poate fi determinat astfel:

$$\omega_H \cong \frac{1}{\sum_{i=1}^m R_{iO} C_i}$$

$R_{iO}$  reprezinta rezistenta echivalenta dintre terminalele condensatorului  $C_i$ , considerand toate celelalte condensatoare considerate a fi circuite deschise.

$\omega_H$  depinde de toate condensatoarele din modelul de semnal mic si inalta frecventa al circuitului analizat.

Limitari ale metodei:

- nu ofera informatii legate de polii ne-dominanti
- nu ofera informatii asupra zerourilor circuitului analizat

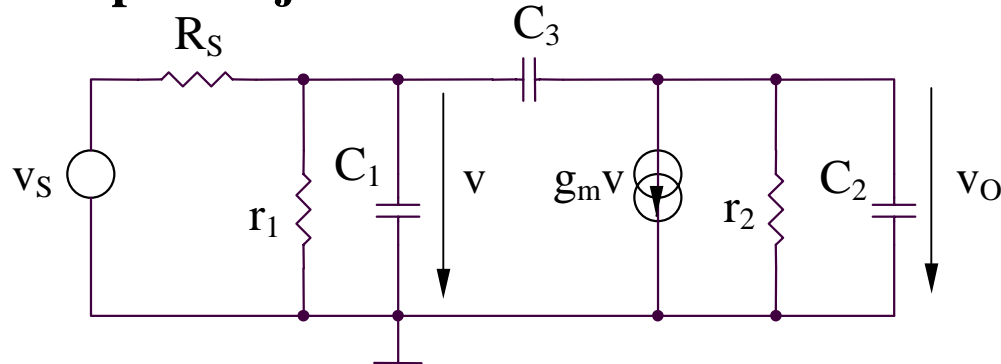
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### B. Metoda constantelor de timp

#### B2. Estimarea $\omega_H$ utilizand metoda constantelor de timp de gol (continuare)

Exemple: etajele emitor comun/sursa comuna



$$\omega_H \cong \frac{1}{\sum_{i=1}^m R_{i0} C_i}$$

Circuitul contine 3 condensatoare, deci va avea 3 constante de timp:

$$\tau_{10} = C_1 R_{10}$$

$$\tau_{20} = C_2 R_{20}$$

$$\tau_{30} = C_3 R_{30}$$

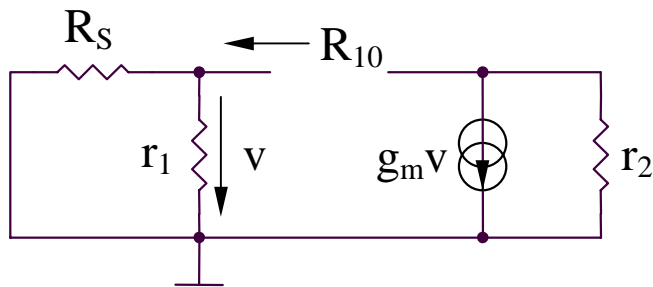
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

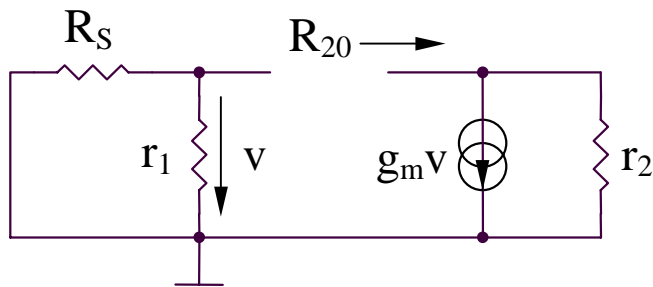
#### B. Metoda constantelor de timp

#### B2. Estimarea $\omega_H$ utilizand metoda constantelor de timp de gol (continuare)

Exemple: etajele emitor comun/sursa comuna



$$R_{10} = r_1 // R_S \Rightarrow \tau_{10} = C_1 (r_1 // R_S)$$



$$R_{20} = r_2 \Rightarrow \tau_{20} = C_2 r_2$$

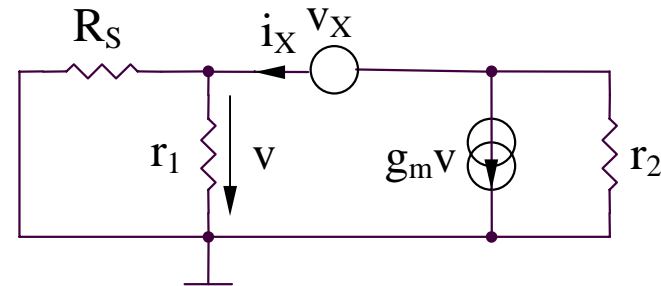
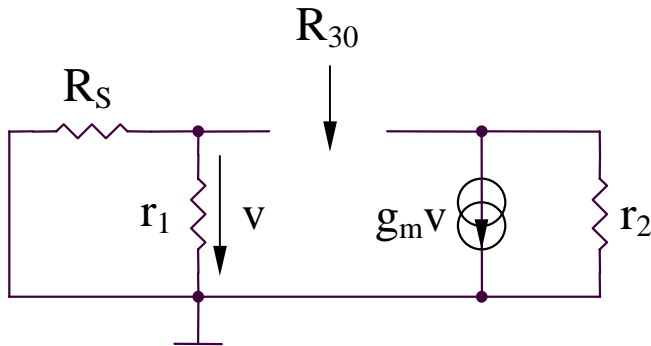
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### B. Metoda constantelor de timp

#### B2. Estimarea $\omega_H$ utilizand metoda constantelor de timp de gol (continuare)

Exemple: etajele emitor comun/sursa comuna



$$\begin{cases} v_X = i_X (r_1 // R_S) + (i_X + g_m v) r_2 \\ v = i_X (r_1 // R_S) \end{cases} \Rightarrow R_{30} = \frac{v_X}{i_X} = r_1 // R_S + r_2 + g_m r_2 (r_1 // R_S)$$

$$\Rightarrow \tau_{30} = R_{30} C_3 = C_3 [r_1 // R_S + r_2 + g_m r_2 (r_1 // R_S)]$$

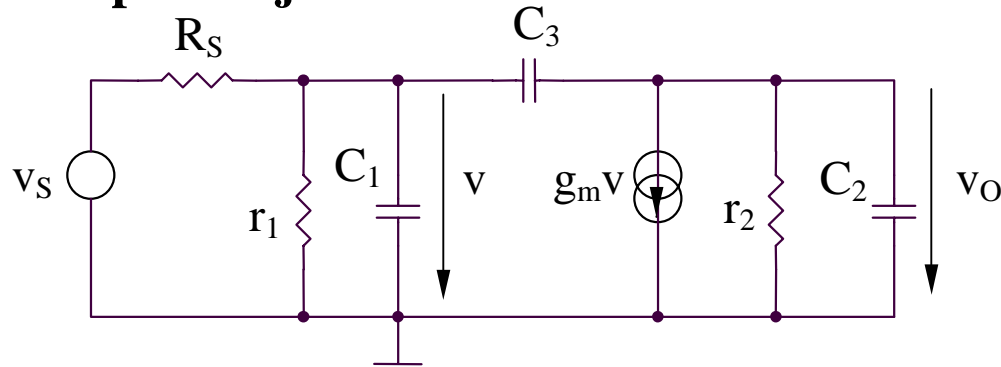
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### B. Metoda constantelor de timp

#### B2. Estimarea $\omega_H$ utilizand metoda constantelor de timp de gol (continuare)

Exemple: etajele emitor comun/sursa comuna



Se obtine:

$$\omega_H = \frac{1}{\tau_{10} + \tau_{20} + \tau_{30}} = \frac{1}{C_1(r_1 // R_S) + C_2 r_2 + C_3 [r_1 // R_S + r_2 + g_m r_2 (r_1 // R_S)]}$$

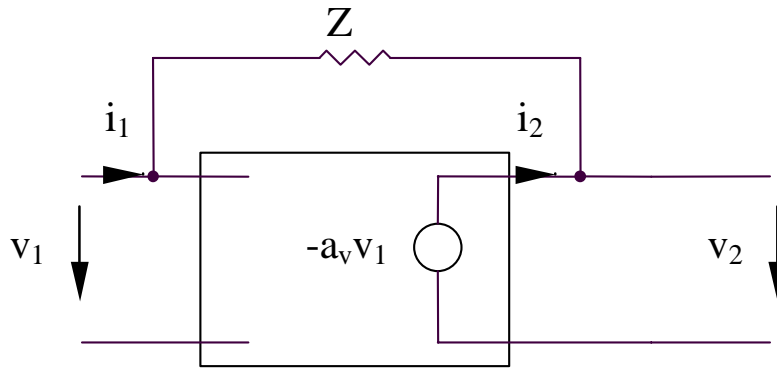
Pozitia zerorului  $\omega_Z$  se obtine din conditia ca semnalul prin condensatorul  $C_3$  sa anuleze semnalul prin sursa de curent controlata in tensiune:

$$\frac{v}{1 / \omega_Z C_3} = g_m v \Rightarrow \omega_Z = \frac{g_m}{C_3}$$

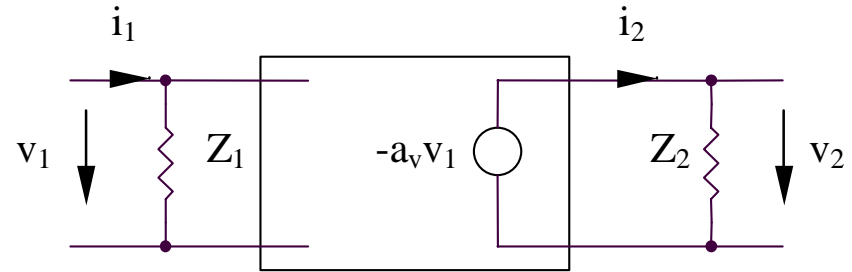
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### C. Teorema lui Miller



(a)



(b)

$$i_1 = \frac{v_1 - v_2}{Z} = \frac{v_1 + a_v v_1}{Z} = \frac{(1 + a_v)v_1}{Z}$$

$$i_2 = \frac{v_2 - v_1}{Z} = -\frac{(1 + a_v)v_1}{Z}$$

$$i_1 = \frac{v_1}{Z_1}$$

$$i_2 = \frac{v_2}{Z_2} = -\frac{a_v v_1}{Z_2}$$

$$\Rightarrow Z_1 = \frac{Z}{1 + a_v} \ll Z; \quad Z_2 = Z \frac{a_v}{1 + a_v} \cong Z$$

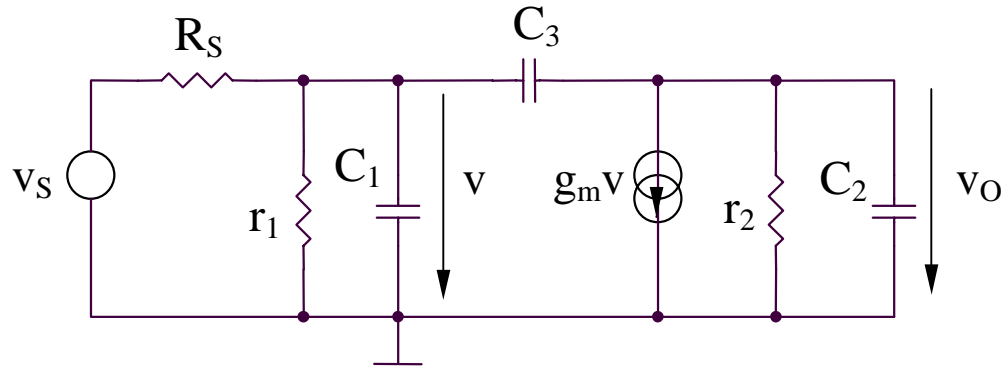


## 7.2. Raspunsul in frecventa al amplificatoarelor

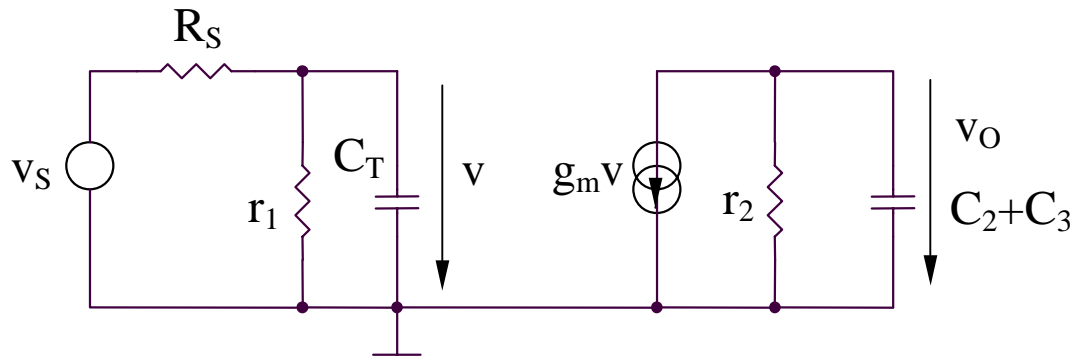
### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### C. Teorema lui Miller

Exemple: etajele emitor comun/sursa comuna



Schema echivalenta (utilizand teorema lui Miller)



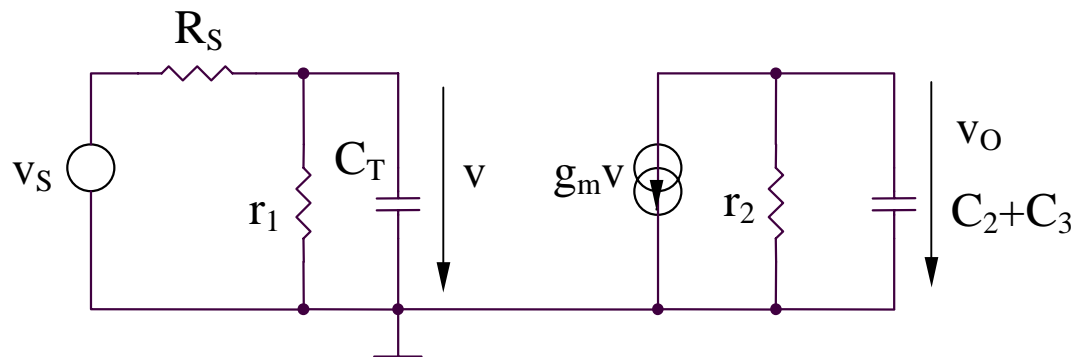
$$C_T = C_1 + C_3(1 + g_m r_2)$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.3. Analiza raspunsului in frecventa al amplificatoarelor elementare

#### C. Teorema lui Miller

**Exemple: etajele emitor comun/sursa comuna**



#### **Concluzii:**

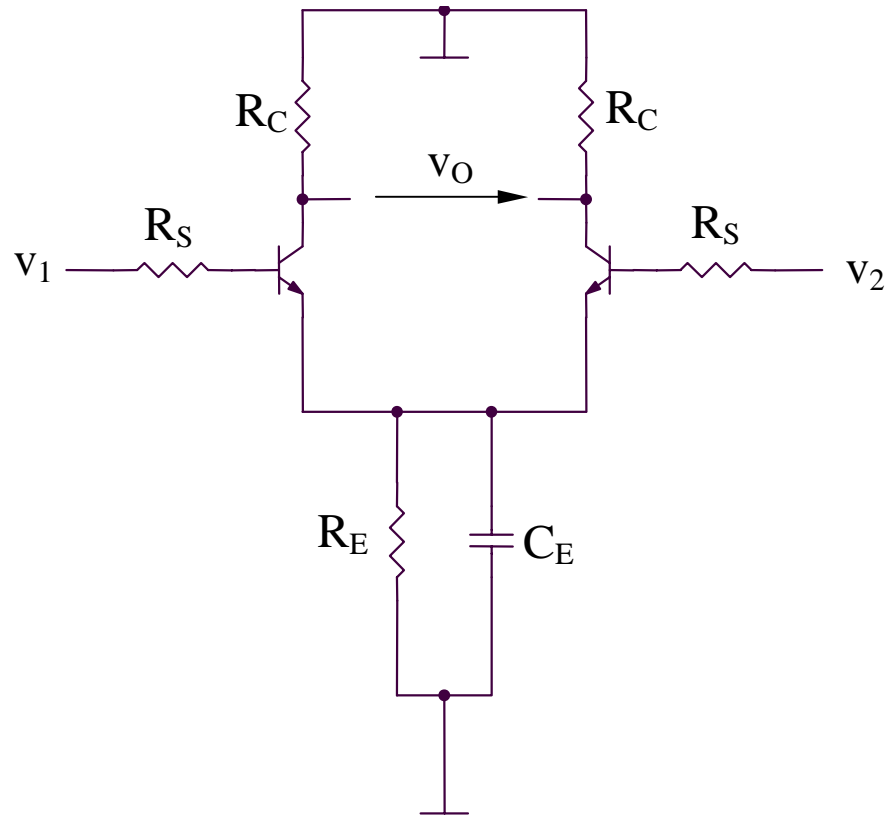
- aplicarea teoremei lui Miller conduce la obtinerea aceleiasi constante de timp dominante ca si in cazul utilizarii metodei constantelor de timp de gol:

$$\tau \cong C_3 g_m r_2 (r_1 // R_S)$$

- metoda poate fi aplicata numai pentru amplificatoare inversoare avand un condensator cuplat intre intrare si iesire

## 7.2. Raspunsul in frecventa al amplificatoarelor

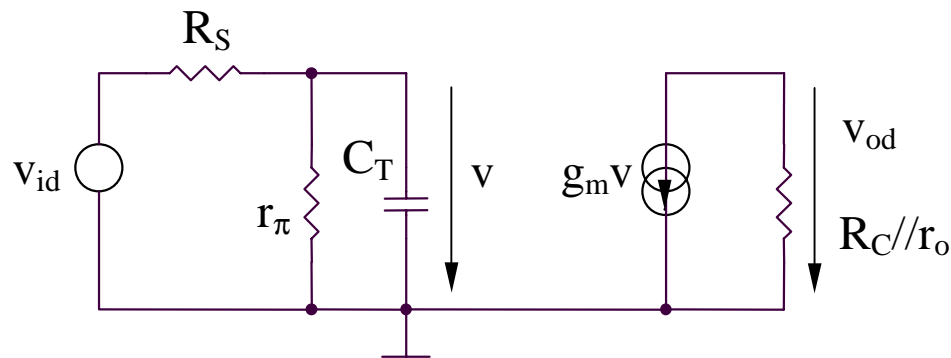
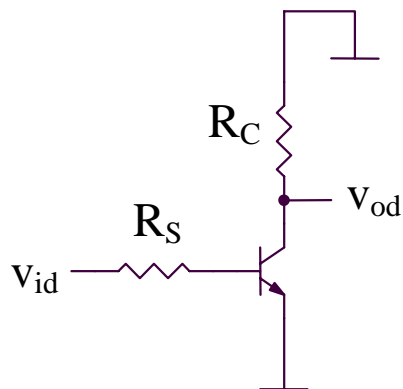
### 7.2.4. Analiza raspunsului in frecventa al amplificatorului diferential



## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.4. Analiza raspunsului in frecventa al amplificatorului diferential

#### Functionarea pe mod differential (MD)



$$A_{dd}(j\omega) = \frac{v_{od}}{v_{id}}(j\omega) = -g_m (R_C // r_o) \frac{r_\pi}{r_\pi + R_S} \frac{1}{1 + j\omega C_T (r_\pi // R_S)}$$

$$A_{dd}(j\omega) = \frac{A_{dd0}}{1 + j \frac{\omega}{\omega_{H(MD)}}}$$

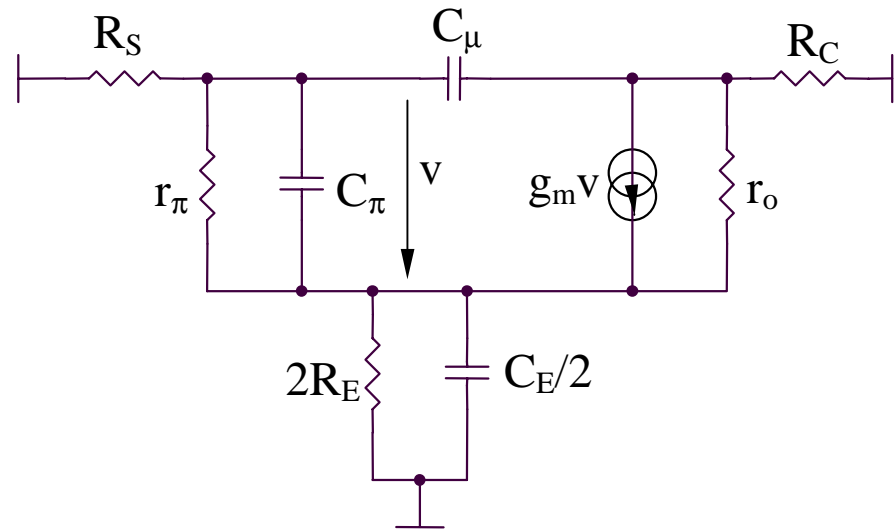
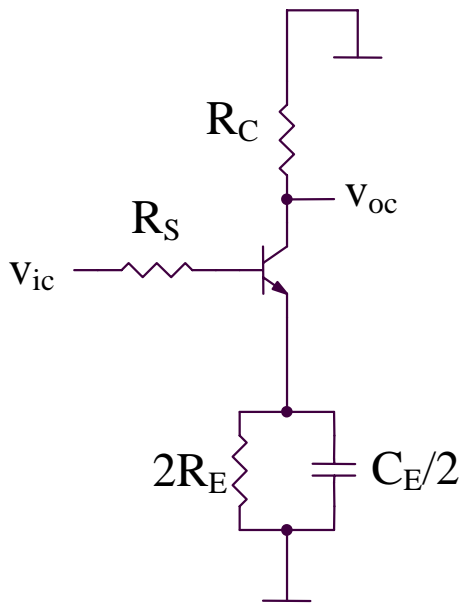
$$A_{dd0} = -g_m (R_C // r_o) \frac{r_\pi}{r_\pi + R_S}$$

$$\omega_{H(MD)} = \frac{1}{(r_\pi // R_S) C_T} = \frac{1}{(r_\pi // R_S) \{C_\pi + C_\mu [1 + g_m (R_C // r_o)]\}}$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.4. Analiza raspunsului in frecventa al amplificatorului diferential

#### Functionarea pe mod comun (MC)



Aplicand metoda constantelor de timp de gol se obtine:

$$\omega_{H(MC)} = \frac{1}{\tau_{E0} + \tau_{\pi0} + \tau_{\mu0}}$$

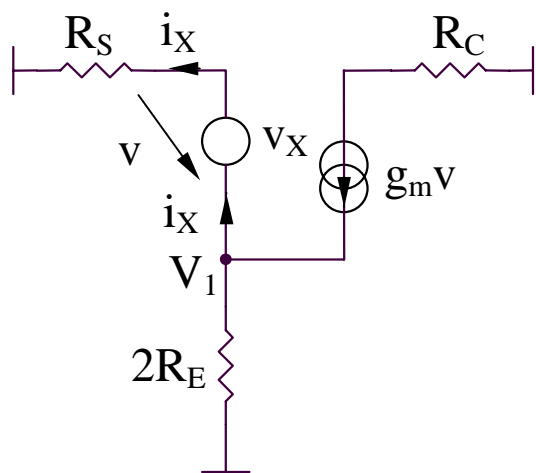
## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.4. Analiza raspunsului in frecventa al amplificatorului diferential

Functionarea pe mod comun (MC)

$$\tau_{E0} = \frac{C_E}{2} \left( 2R_E // \frac{r_\pi + R_S}{\beta + 1} \right)$$

Circuit pentru calculul  $\tau_{\pi 0}$



$$\begin{aligned} v_X &= i_X R_S - V_1 \\ \frac{V_1}{2R_E} &= -i_X + g_m v_X \end{aligned} \quad \Rightarrow \quad \frac{i_X R_S - v_X}{2R_E} = -i_X + g_m v_X$$

Se obtin:

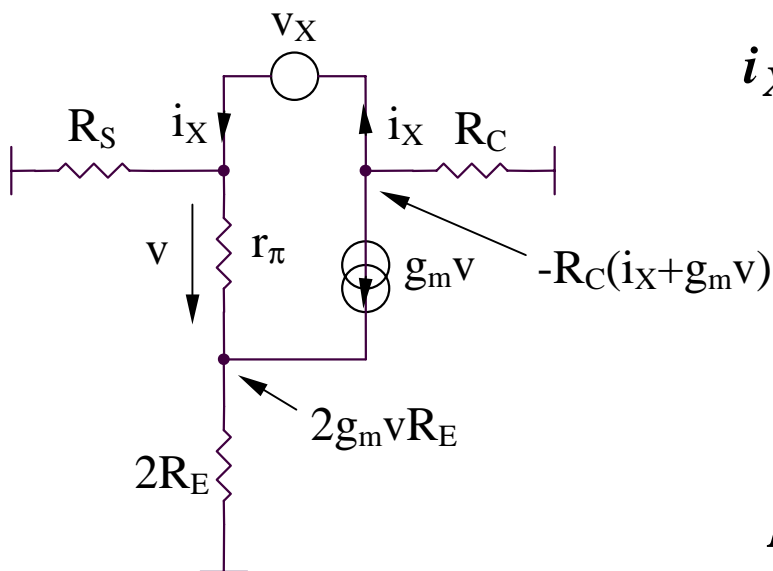
$$R_{\pi 0} = \frac{v_X}{i_X} // r_\pi = \frac{2R_E + R_S}{1 + 2g_m R_E} // r_\pi \quad \tau_{\pi 0} = C_\pi \left[ \frac{2R_E + R_S}{1 + 2g_m R_E} // r_\pi \right]$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.4. Analiza raspunsului in frecventa al amplificatorului diferential

#### Functionarea pe mod comun (MC)

Circuit pentru calculul  $\tau_{\mu 0}$



$$i_X = \frac{v}{r_\pi} + \frac{v + 2g_m R_E v}{R_S} = \frac{R_S + r_\pi(1 + 2g_m R_E)}{r_\pi R_S} v$$

$$v_X = (v + 2g_m R_E v) + R_C (i_X + g_m v)$$

Rezulta:

$$R_{\mu 0} = \frac{v_X}{i_X} \cong R_S + R_C + \frac{g_m}{1 + 2g_m R_E} R_S R_C$$

$$\tau_{\mu 0} \cong C_\mu \left( R_S + R_C + \frac{g_m}{1 + 2g_m R_E} R_S R_C \right)$$

## 7.2. Raspunsul in frecventa al amplificatoarelor

### 7.2.4. Analiza raspunsului in frecventa al amplificatorului diferential

#### Functionarea pe mod comun (MC)

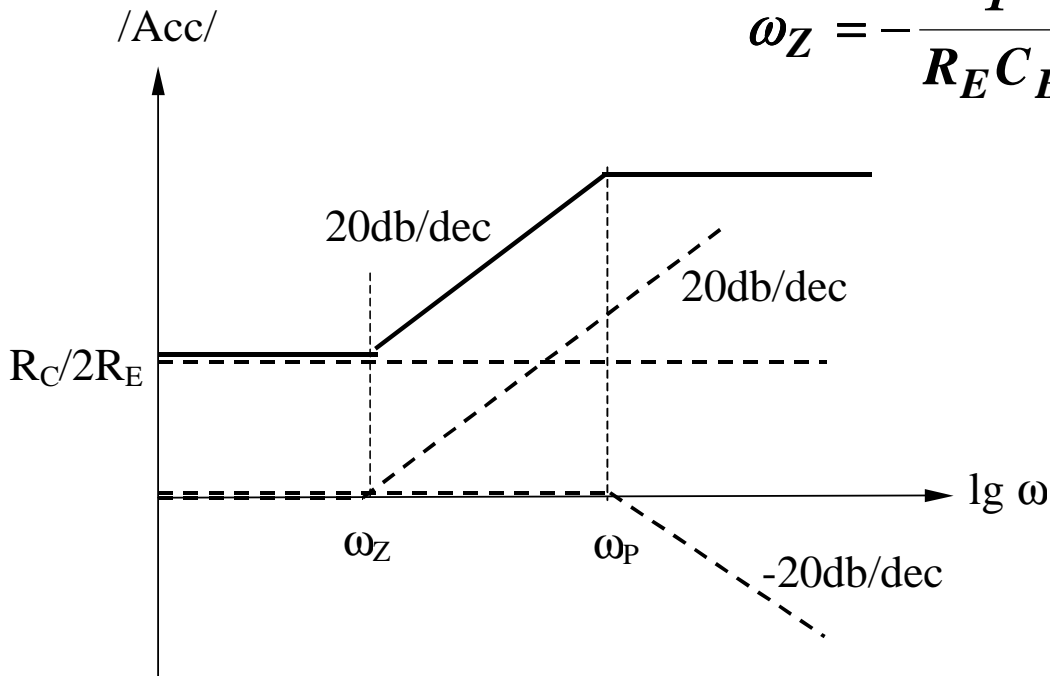
##### Observatie

Amplificarea de mod comun are expresia aproximativa:

$$A_{cc} \cong -\frac{R_C}{2R_E // (2 / j\omega C_E)} = -\frac{R_C}{2R_E} (1 + j\omega C_E R_E)$$

deci condensatorul  $C_E/2$  introduce un zero la  $\omega_Z$  avand expresia:

$$\omega_Z = \frac{1}{R_E C_E}$$



$A_{cc}$  creste o data cu cresterea frecventei