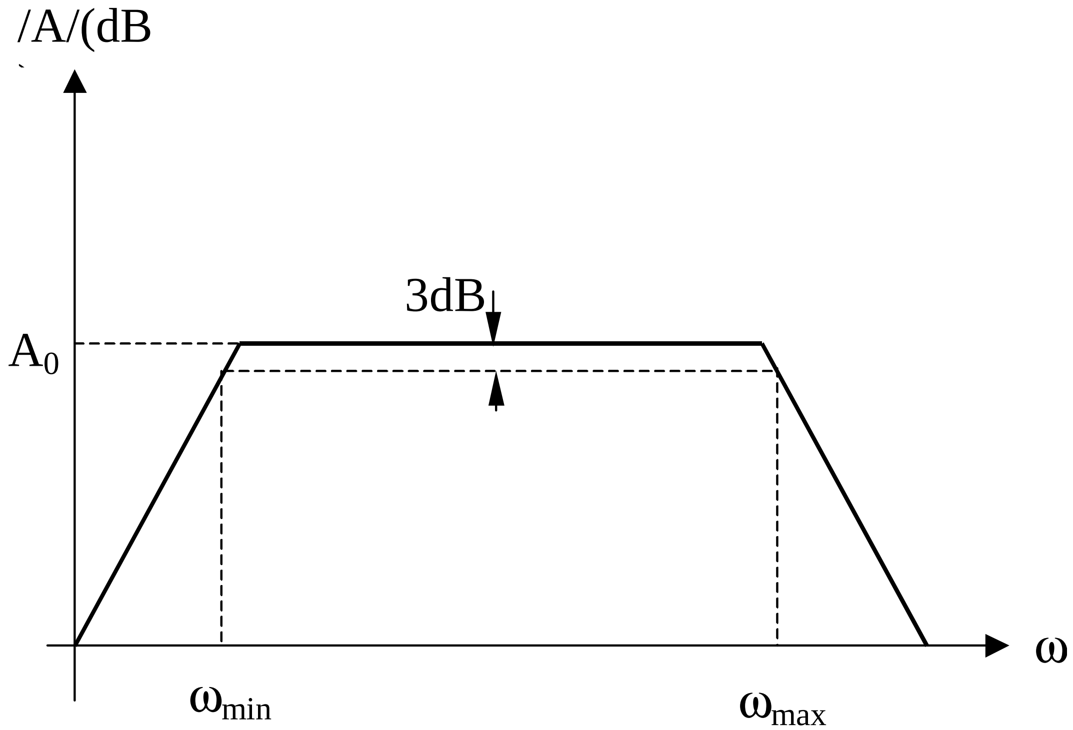
Chapter 6
Frequency response of circuits. Stability

6.1. The frequency response of elementary functions

6.1.1. The frequency bandwidth

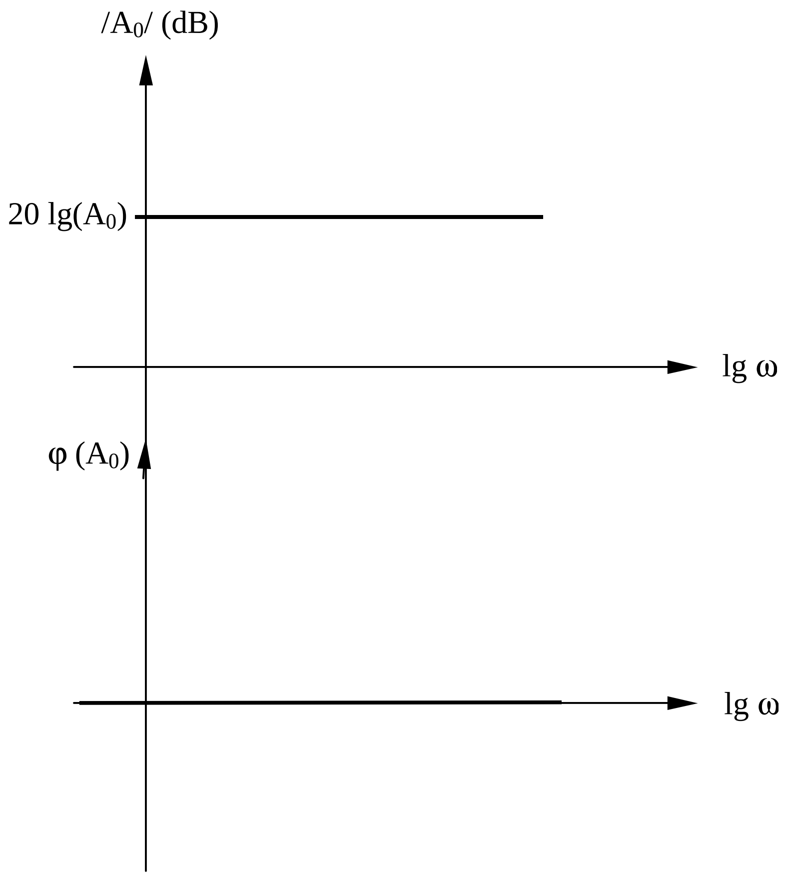
6.1.1. The frequency bandwidth



6.1.2. The frequency response of elementary functions

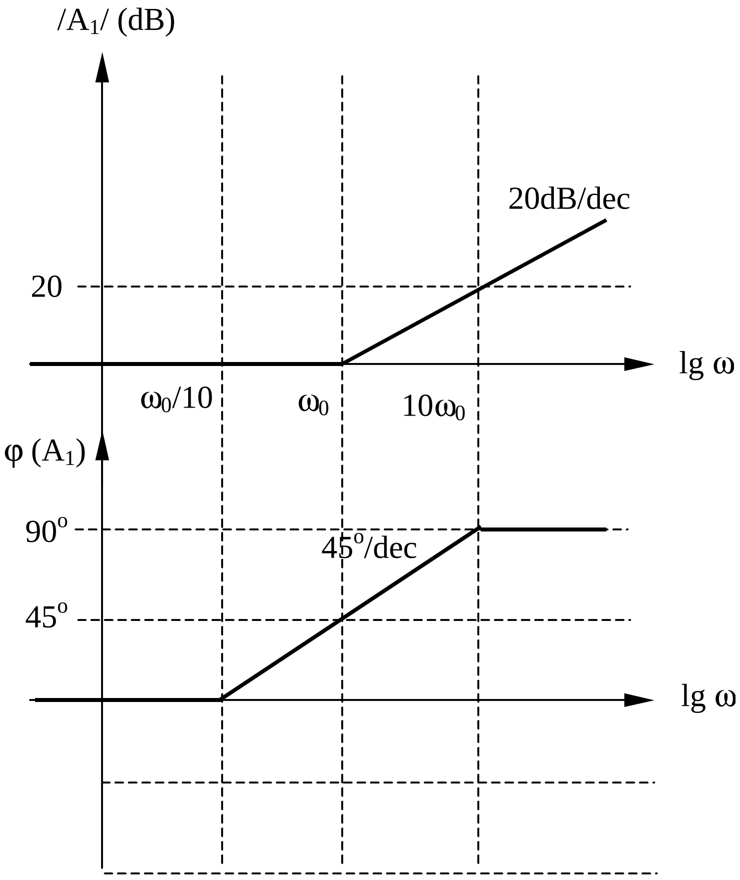
6.1.2. The frequency response of elementary functions

A constant



$$A_0 = ct.$$

A simple zero



$$A_1 = 1 + j\frac{\omega}{\omega_0}$$

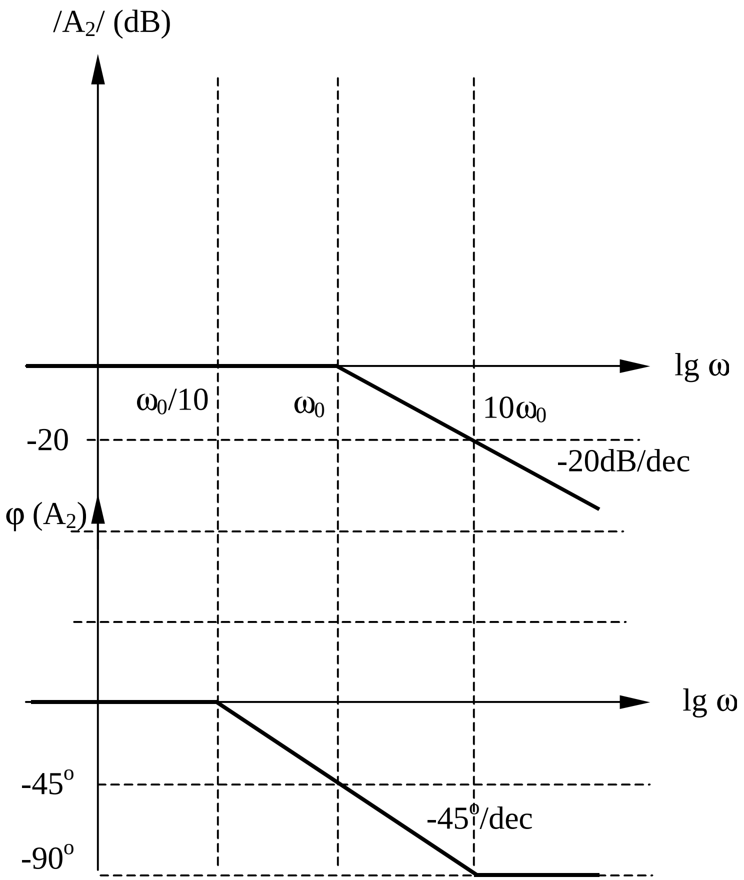
$$|A_1| = 20\,\mathrm{lg}\left[\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\omega << \omega_0 \Rightarrow |A_1| \to 0$$

$$\omega >> \omega_0 \Rightarrow |A_1| \to 20\,\mathrm{lg}\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi(A_1) = arctg\left(\frac{\omega}{\omega_0}\right)$$

A simple pole



$$A_2 = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

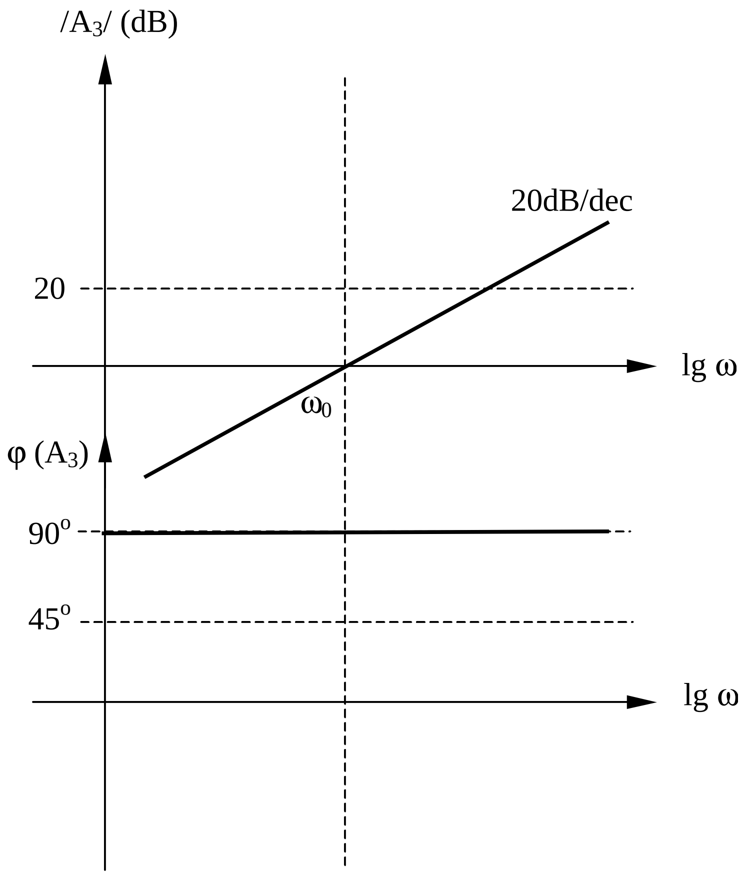
$$\left|A_2\right| = -20\,\mathrm{lg}\left[\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\omega << \omega_0 \Rightarrow \left|A_2\right| \rightarrow 0$$

$$\omega >> \omega_0 \Rightarrow \left|A_2\right| \rightarrow -20\,\mathrm{lg}\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi(A_2) = -arctg\left(\frac{\omega}{\omega_0}\right)$$

A simple zero in origin

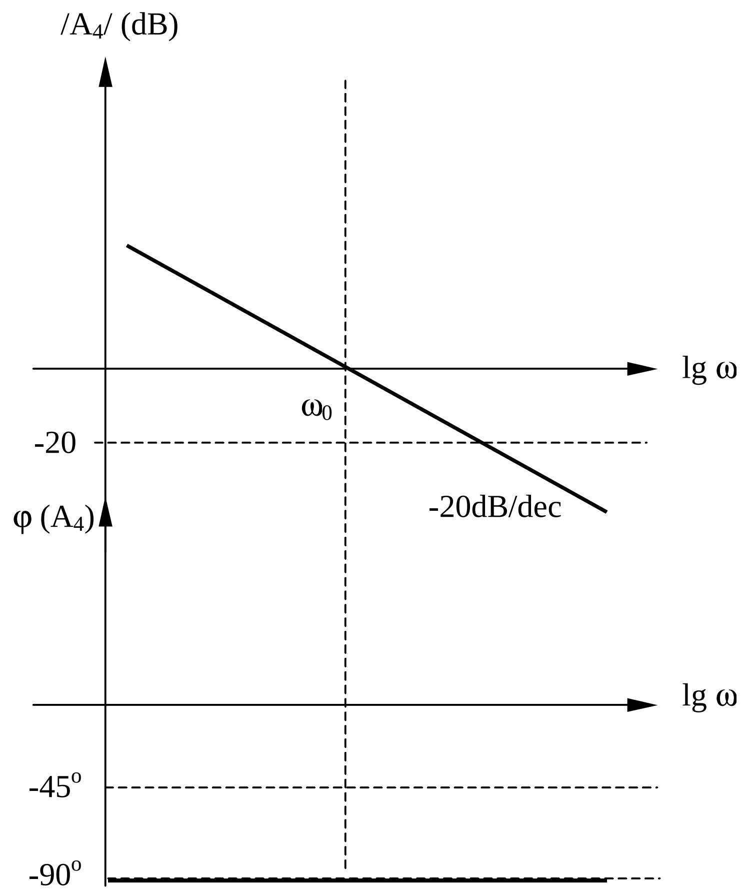


$$A_3 = j\frac{\omega}{\omega_0}$$

$$\left|A_3\right| = 20\,\mathbf{lg}\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi\left(A_3\right) = 90^o$$

A simple pole in origin

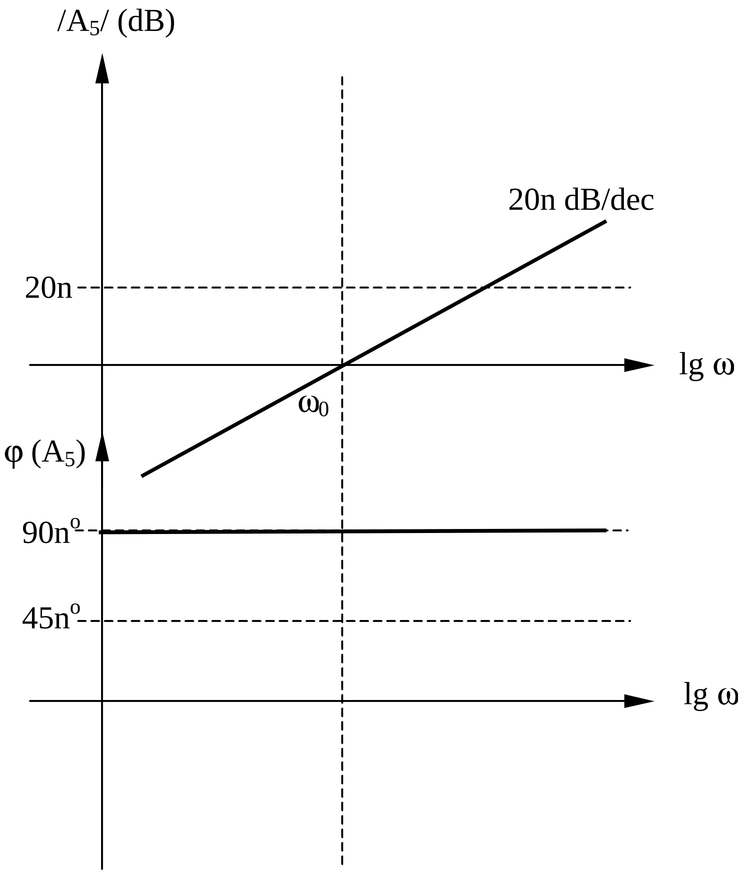


$$A_4 = \frac{1}{j\frac{\omega}{\omega_0}}$$

$$|A_4| = -20\,\mathbf{lg}\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi(A_4) = -90^o$$

A multiple zero in origin

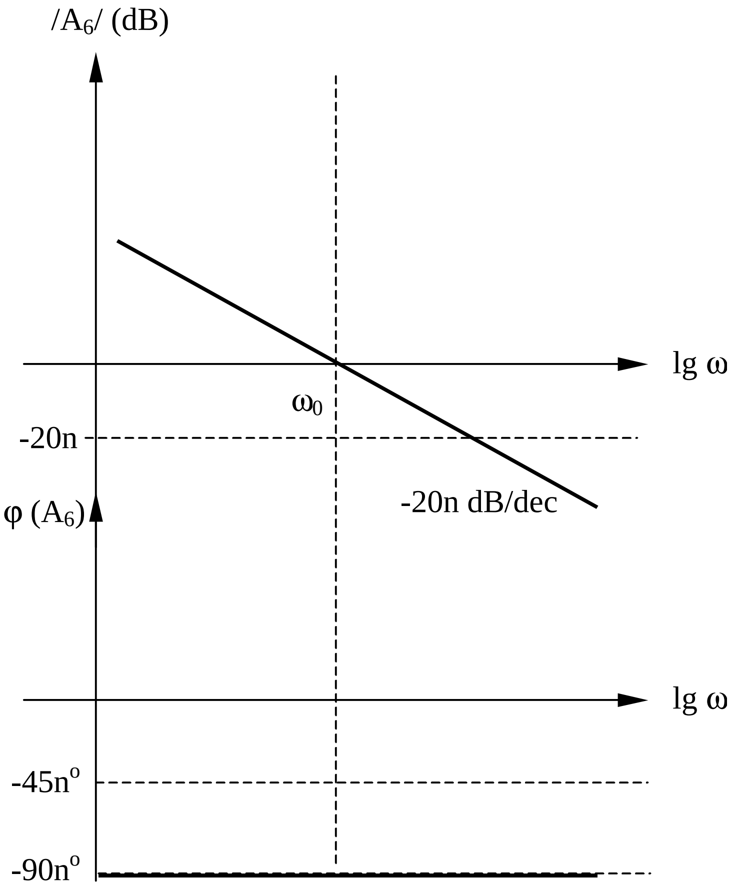


$$A_5 = \left(j\frac{\omega}{\omega_0} \right)^n$$

$$\left| A_5 \right| = 20 \times n \lg \left(\frac{\omega}{\omega_0} \right)$$

$$\varphi\left(A_5 \right) = n \times 90^o$$

A multiple pole in origin

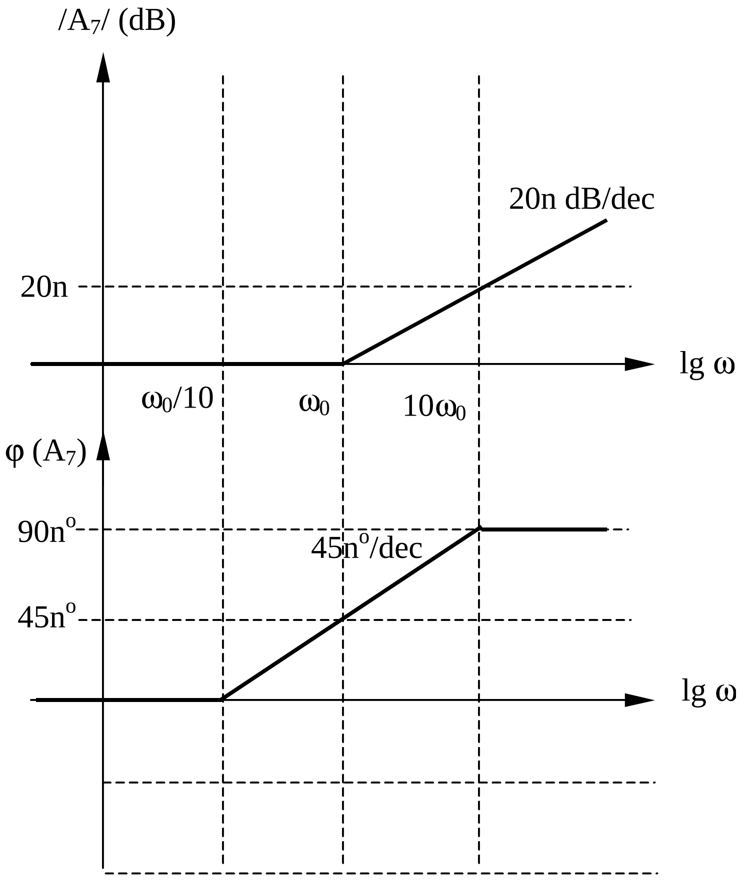


$$A_6 = \frac{1}{\left(j\frac{\omega}{\omega_0}\right)^n}$$

$$\left|A_6\right| = -20 \times n \,\mathbf{lg}\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi\left(A_6\right) = -n \times 90^o$$

A multiple zero



$$A_7 = \left(1 + j\frac{\omega}{\omega_0}\right)^n$$

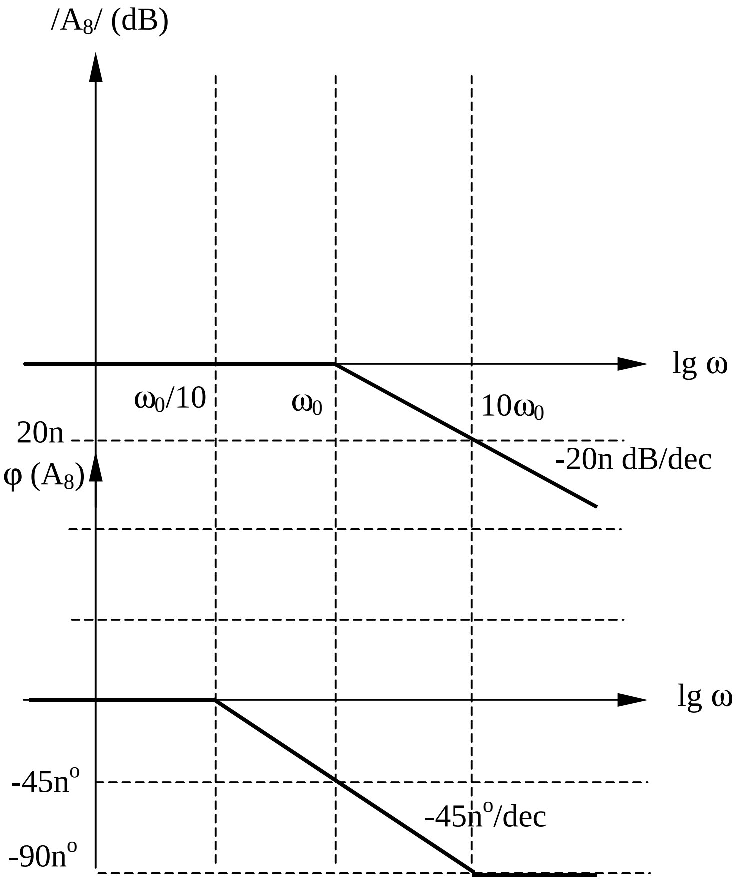
$$|A_7| = -20 \times n \lg\left[\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\omega << \omega_0 \Rightarrow |A_7| \to 0$$

$$\omega >> \omega_0 \Rightarrow |A_7| \to -20 \times n \lg\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi(A_7) = -n \times arctg\left(\frac{\omega}{\omega_0}\right)$$

A multiple pole



$$A_8 = \frac{1}{\left(1 + j\frac{\omega}{\omega_0}\right)^n}$$

$$\left|A_8\right| = -20 \times n\,\mathrm{lg}\left[\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right]$$

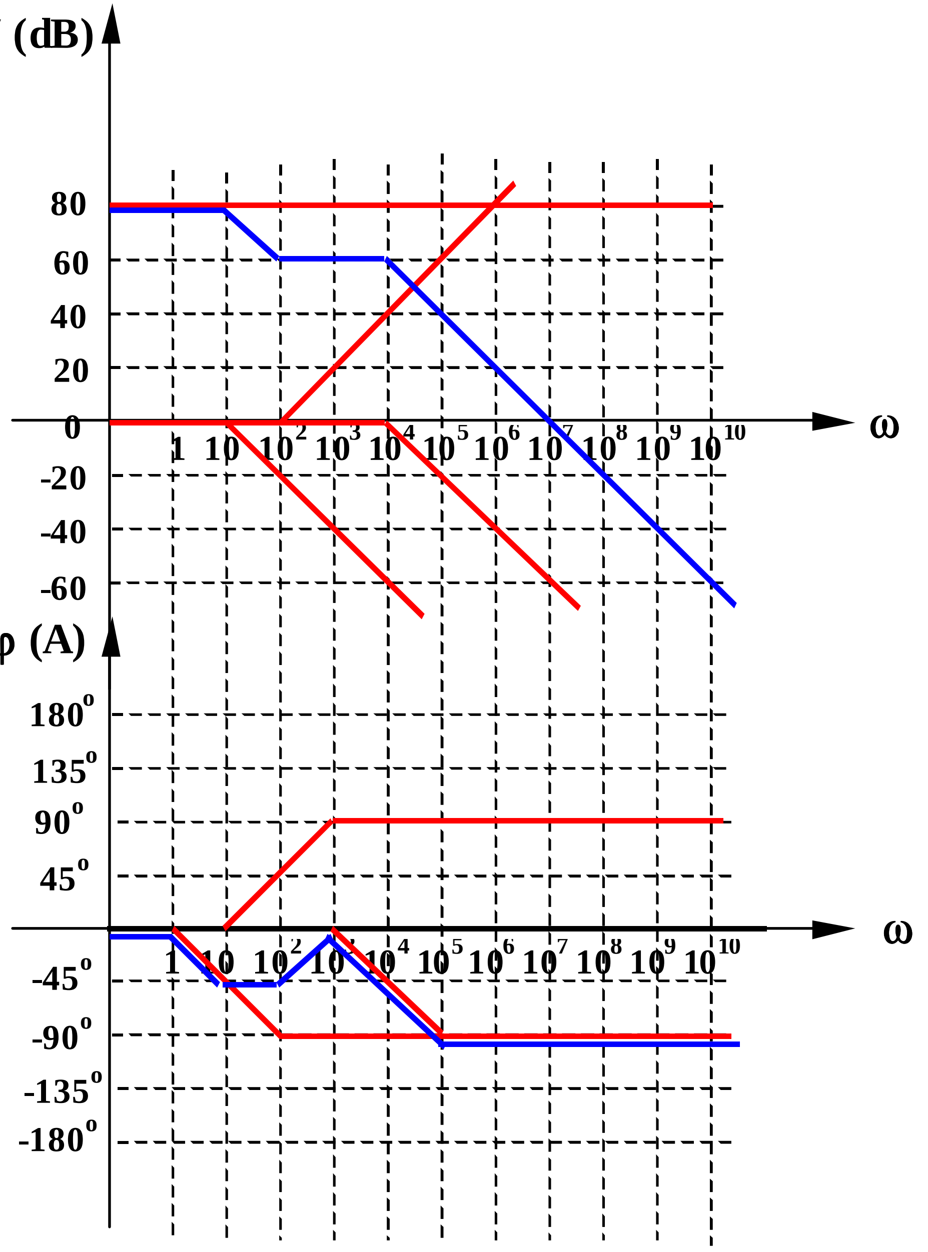
$$\omega << \omega_0 \Rightarrow \left|A_8\right| \to 0$$

$$\omega >> \omega_0 \Rightarrow \left|A_8\right| \to -20 \times n\,\mathrm{lg}\left(\frac{\omega}{\omega_0}\right)$$

$$\varphi(A_8) = -n \times arctg\left(\frac{\omega}{\omega_0}\right)$$

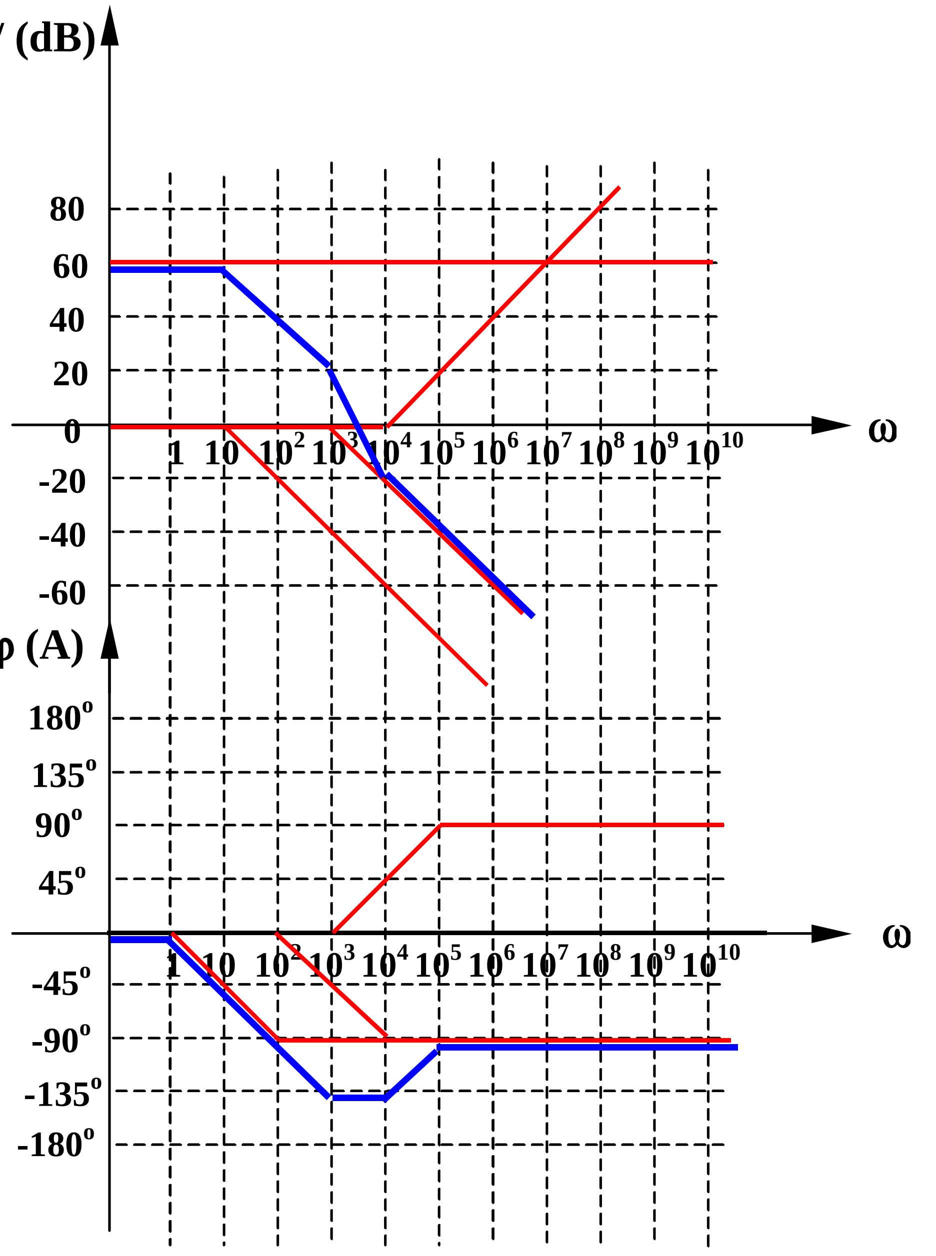
Example 1

$$A(j\omega) = 10^4 \frac{1 + j\dfrac{\omega}{10^2}}{\left(1 + j\dfrac{\omega}{10}\right)\left(1 + j\dfrac{\omega}{10^4}\right)}$$

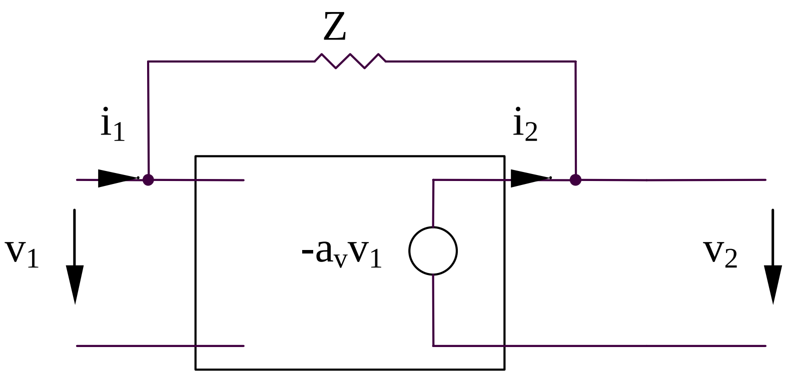


Example 2

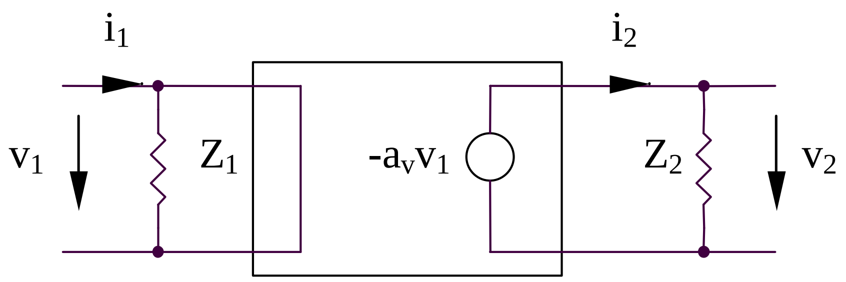
$$A(j\omega) = 10^3 \frac{1 + j\dfrac{\omega}{10^4}}{\left(1 + j\dfrac{\omega}{10}\right)\left(1 + j\dfrac{\omega}{10^3}\right)}$$



6.1.3. Miller theorem



(a)



(b)

$$i_1 = \frac{v_1 - v_2}{Z} = \frac{v_1 + a_v v_1}{Z} = \frac{(1+a_v)v_1}{Z}$$

$$i_2 = \frac{v_2 - v_1}{Z} = -\frac{(1+a_v)v_1}{Z}$$

$$i_1 = \frac{v_1}{Z_1}$$

$$i_2 = \frac{v_2}{Z_2} = -\frac{a_v v_1}{Z_2}$$

$$\Rightarrow Z_1 = \frac{Z}{1+a_v} << Z; \; Z_2 = Z\frac{a_v}{1+a_v} \cong Z$$

6.1.3. Miller theorem

6.2. Amplifiers with reaction

6.2.1. The block diagram of the amplifier with reaction

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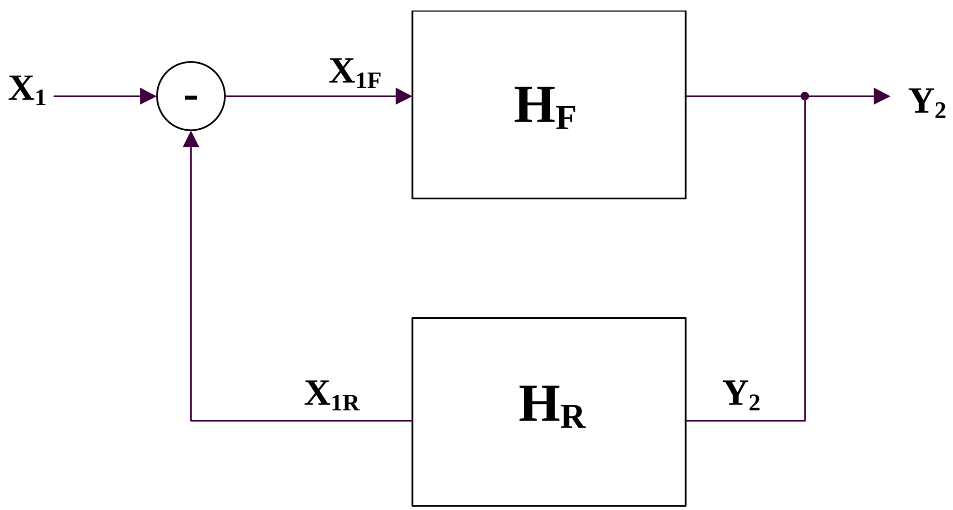

$$Y_2 = H_F X_{1F}$$

$$X_{1R} = H_R Y_2$$

$$X_{1F} = X_1 - X_{1R} = X_1 - H_F H_R X_{1F}$$

X_1 , Y_2 **are currents/voltages**

The global gain: $H = \dfrac{Y_2}{X_1} = \dfrac{H_F}{1 + H_R H_F}$

6.2.2. Types of reaction

- Positive reaction: $|H| > |H_F|$ $\quad |1 + H_F H_R| < 1$

- Negative reaction: $|H| < |H_F|$ $\quad |1 + H_F H_R| > 1$

Particular case: <u>strong negative reaction</u>

Defining loop transmission: $T = \dfrac{X_{1R}}{X_{1F}} = H_F H_R >> 1 \quad \left(|H| << |H_F|\right)$

it results $H\big|_{T>>1} = \dfrac{H_F}{H_F H_R} = \dfrac{1}{H_R}$ **- independent on amplifier**

Conclusion: ***for strong negative reaction, the gain with reaction is function only on reaction***

6.2.2. Types of reaction

6.3. Reaction effects

6.3.1. Amplifier de-sensitivity

6.3.1. Amplifier de-sensitivity

$$\frac{dH}{dH_F} = \frac{d}{dH_F}\left(\frac{H_F}{1+H_F H_R}\right) = \frac{1}{(1+H_F H_R)^2}$$

$$\left|\frac{dH}{H}\right| = \frac{1}{|1+H_R H_F|}\left|\frac{dH_F}{H_F}\right| = \frac{1}{|F|}\left|\frac{dH_F}{H_F}\right|$$

$$F = 1 + H_R H_F = 1 + T$$

(reaction factor)

6.3.2. Distortion reduction

6.3.2. Distortion reduction

The reaction reduces the effect of distortions.

6.3.3. The improving of frequency response

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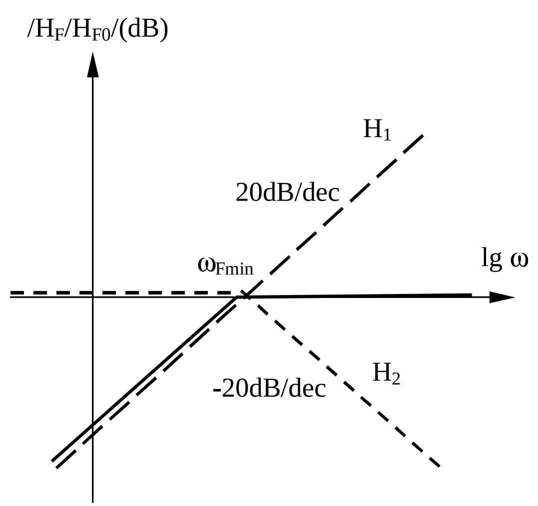
For ω_{min}

Supposing that the direct amplifier is characterized by a first-order function:

$$H_F(j\omega) = H_{F0}\frac{\frac{j\omega}{\omega_{F\,min}}}{1+\frac{j\omega}{\omega_{F\,min}}}$$



and that we have a constant negative reaction H_{R0}, it results:

$$H(j\omega) = \frac{H_F(j\omega)}{1+H_F(j\omega)H_{R0}}$$

resulting:

$$H(j\omega) = \frac{H_{F0}\frac{j\omega}{\omega_{F\,min}}}{1+\frac{j\omega}{\omega_{F\,min}}+H_{F0}H_{R0}\frac{j\omega}{\omega_{F\,min}}}$$

equivalent with:

$$H(j\omega)=\frac{H_{F0}}{1+H_{F0}H_{R0}}\,\frac{\frac{j\omega}{\omega_{F\min}}\left(1+H_{F0}H_{R0}\right)}{1+\frac{j\omega}{\omega_{F\min}}\left(1+H_{F0}H_{R0}\right)}$$

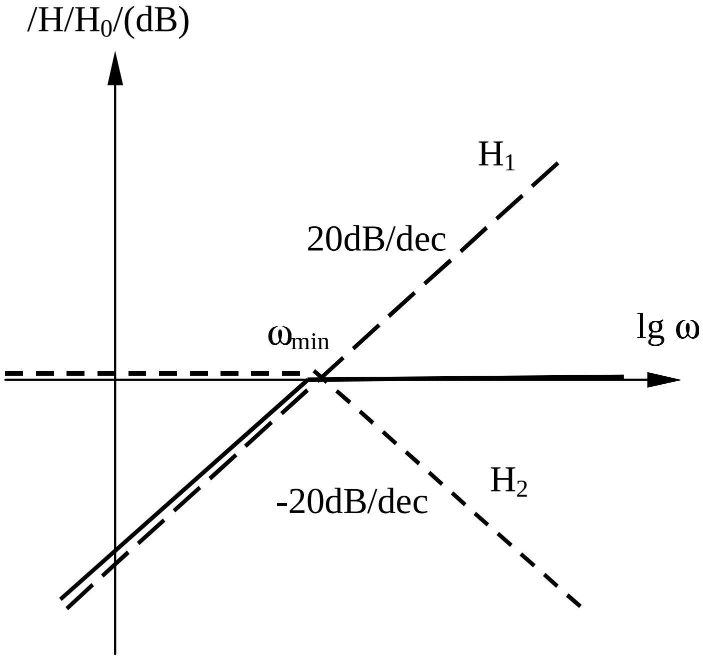
It is possible to find the following form of $H(j\omega)$:

$$H(j\omega)=H_0\,\frac{\frac{j\omega}{\omega_{\min}}}{1+\frac{j\omega}{\omega_{\min}}}$$

where:

$$H_0=\frac{H_{F0}}{1+H_{F0}H_{R0}} \qquad \omega_{\min}=\frac{\omega_{F\min}}{1+H_{F0}H_{R0}}$$

Conclusion: ***The amplifier bandwidth is increased with the same factor of the gain decreasing.***

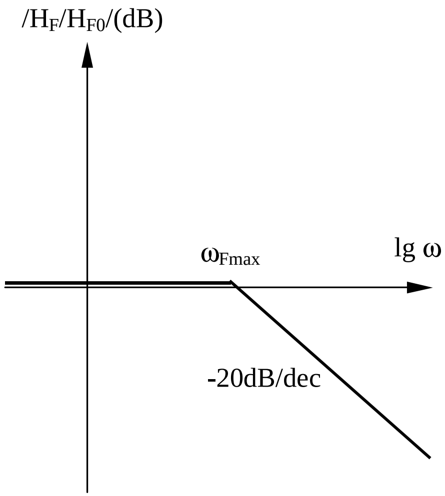


6.3.3. The improving of frequency response

For ω_{max}

Supposing that the direct amplifier is characterized by a first-order function:

$$H_F(j\omega) = \frac{H_{F0}}{1 + \frac{j\omega}{\omega_{F\,max}}}$$



and that we have a constant negative reaction H_{R0}, it results:

$$H(j\omega) = \frac{H_F(j\omega)}{1 + H_F(j\omega) H_{R0}}$$

resulting:

$$H(j\omega) = \frac{H_{F0}}{1 + H_{F0} H_{R0} + \frac{j\omega}{\omega_{F\,max}}}$$

equivalent with:

$$H(j\omega) = \frac{H_{F0}}{1 + H_{F0}H_{R0}} \frac{H_{F0}}{1 + \frac{j\omega}{\omega_{F\max}(1 + H_{F0}H_{R0})}}$$

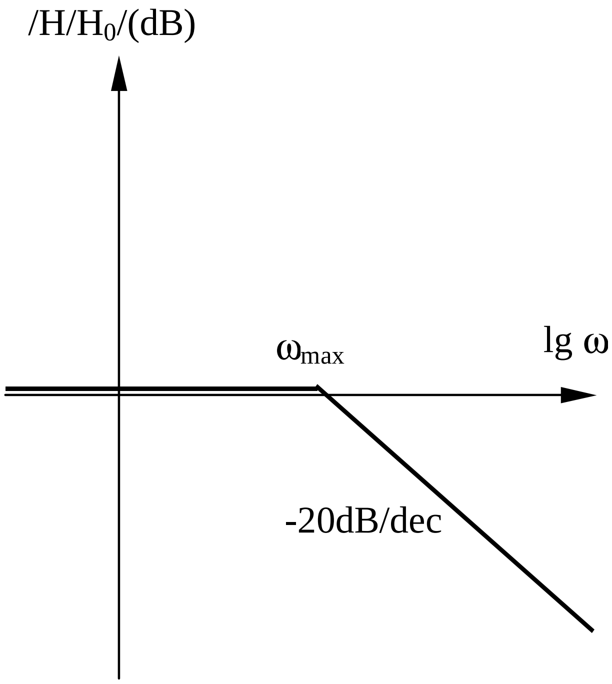
It is possible to find the following form of $H(j\omega)$:

$$H(j\omega) = H_0 \frac{1}{1 + \frac{j\omega}{\omega_{\max}}}$$

where:

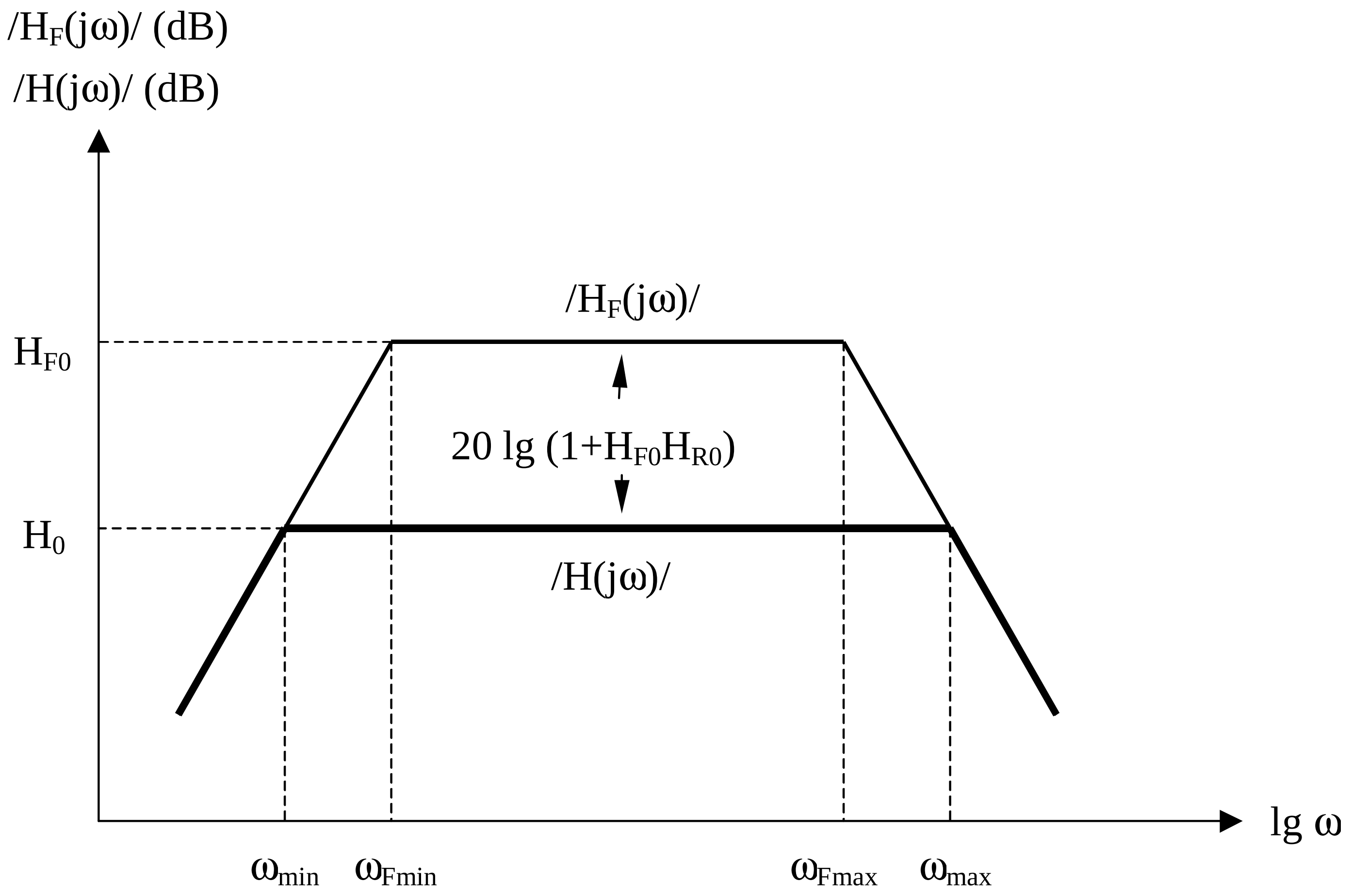
$$H_0 = \frac{H_{F0}}{1 + H_{F0}H_{R0}} \qquad \omega_{\max} = \omega_{F\max}(1 + H_{F0}H_{R0})$$

Conclusion: ***The amplifier bandwidth is increased with the same factor of the gain decreasing.***



6.3.3. The improving of frequency response

Conclusion:



6.3.4. The impact on input/output resistances

6.3.4. The impact on input/output resistances

The reaction changes input/output resistances in such a way that the amplifier with reaction simulates better an ideal amplifier.

$$R_i' = R_i(1+T)$$ **for series reactions**

$$R_i' = R_i(1+T)^{-1}$$ **for parallel reactions**

$$R_o' = R_o(1+T)$$ **for series reactions**

$$R_o' = R_o(1+T)^{-1}$$ **for parallel reactions**

6.4. Circuits stability

6.4.1. Algorithm for evaluating the stability of a circuit

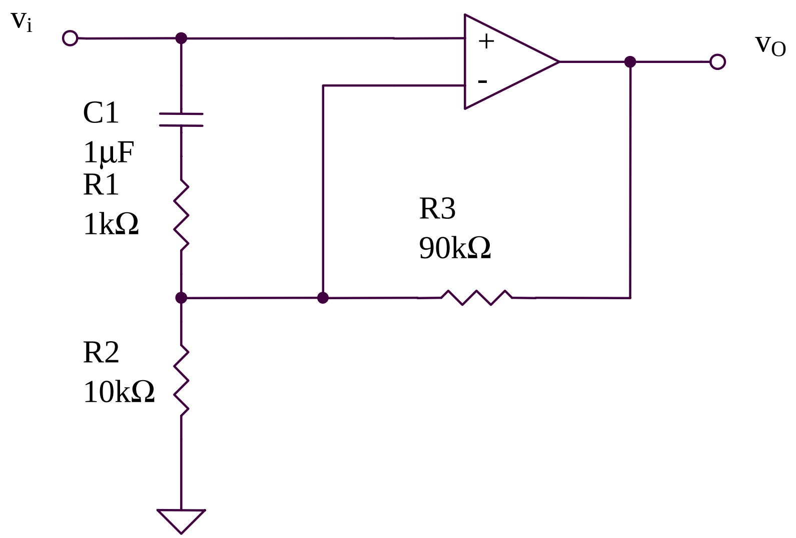
6.4.1. Algorithm for evaluating the stability of a circuit

1. Annulate the input voltage
2. Split the reaction loop in an arbitrary point
3. Apply a test voltage in this point, V_{test}
4. Calculate the “return” voltage in the same point, V_{tr}
5. Compute the Return Ratio $T = V_{tr}/V_{test}$
6. Represent the Bode diagrams for T
7. Represent an horizontal line at -180^{o}
   A. If the horizontal does not intersect the phase graphic, the circuit is stable
   B. If the horizontal intersects the phase graphic in a point A, from A represent a vertical axis which intersects the module diagram in point B
      a. if $/T/_{B} > 0$, the circuit is not stable
      b. if $/T/_{B}$ 0 0, the circuit at the stability limit
      c. if $/T/_{B} < 0$, the circuit is stable. In this case it is possible to determine the phase margin: mark with C the point in which $/T/ = 0$, represent a vertical axis from this point, which will intersect the phase diagram in point D. The phase margin is $\Delta\varphi = 180^{o} + \varphi(D)$

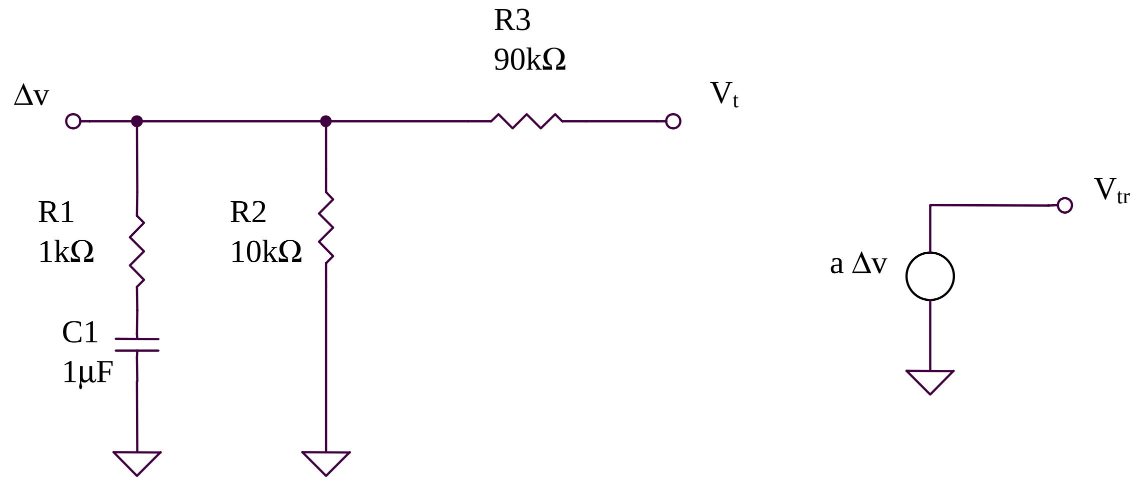
6.4.2. Example 1

6.4.2. Example 1

Evaluate the stability of the following circuit



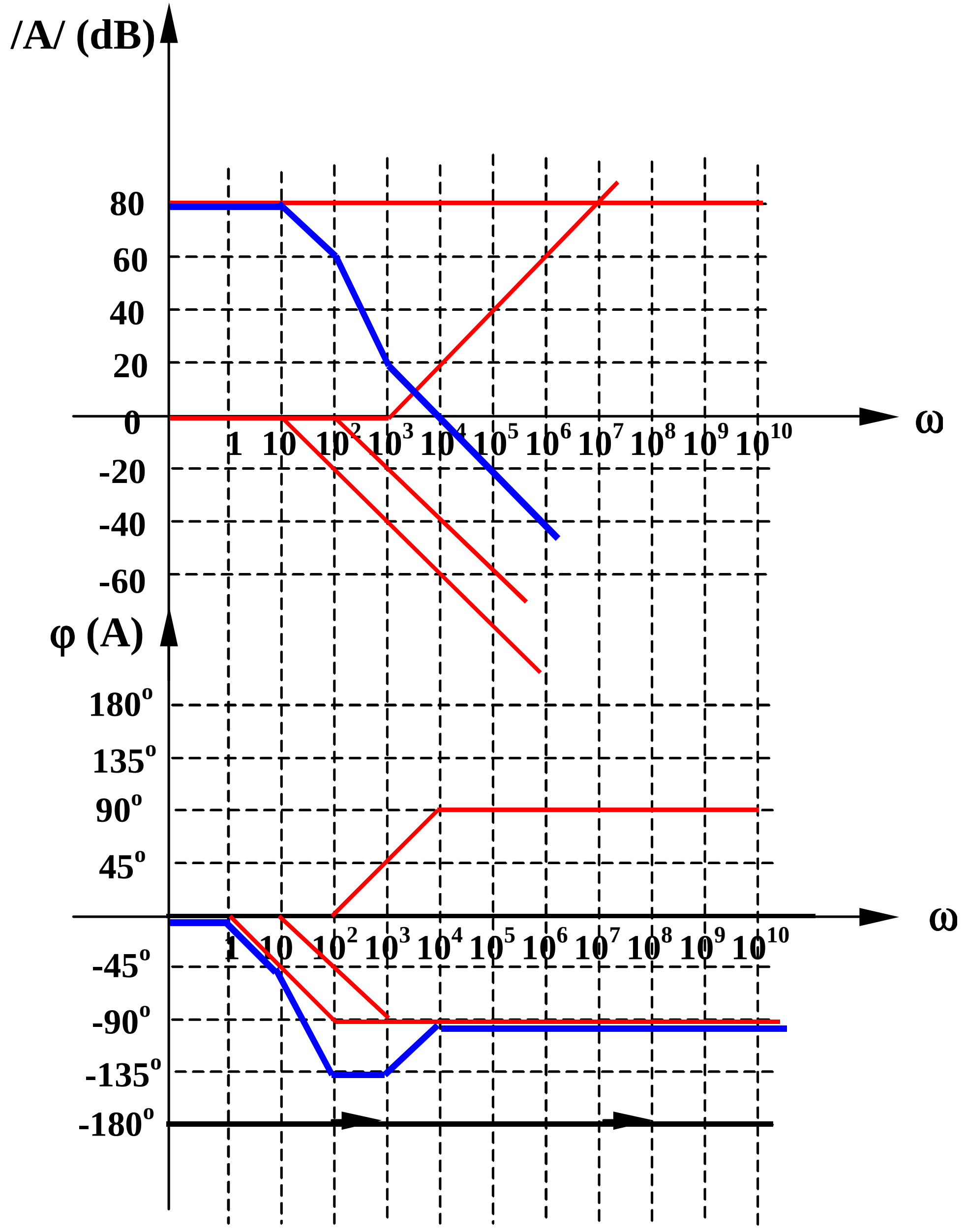
$$a(j\omega)=\frac{10^5}{1+j\frac{\omega}{10}}$$



$$T = \frac{V_{tr}}{V_t} = \frac{a\Delta v}{V_t} = a\frac{R_2 // (R_1 + X_{C1})}{R_2 // (R_1 + X_{C1}) + R_3}$$

$$T = a\frac{\dfrac{R_2(1 + j\omega C_1 R_1)}{1 + j\omega C_1(R_1 + R_2)}}{\dfrac{R_2(1 + j\omega C_1 R_1)}{1 + j\omega C_1(R_1 + R_2)} + R_3} = a\frac{R_2}{R_2 + R_3}\frac{1 + j\omega C_1 R_1}{1 + j\omega C_1[R_1 + (R_2 // R_3)]}$$

$$T = 10^4 \frac{1 + j\dfrac{\omega}{10^3}}{\left(1 + j\dfrac{\omega}{10}\right)\left(1 + j\dfrac{\omega}{10^2}\right)}$$

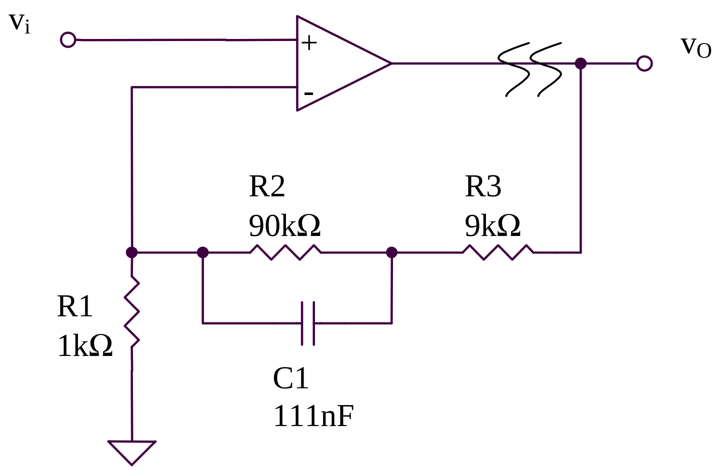


The horizontal line does not intersect the phase diagram, so the circuit is stable.

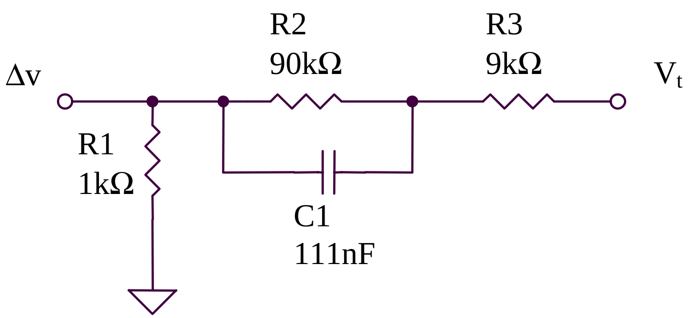
6.4.3. Example 2

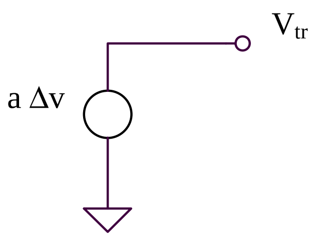
6.4.3. Example 2

Evaluate the stability of the following circuit



$$a(j\omega)=\frac{10^5}{1+j\frac{\omega}{10}}$$

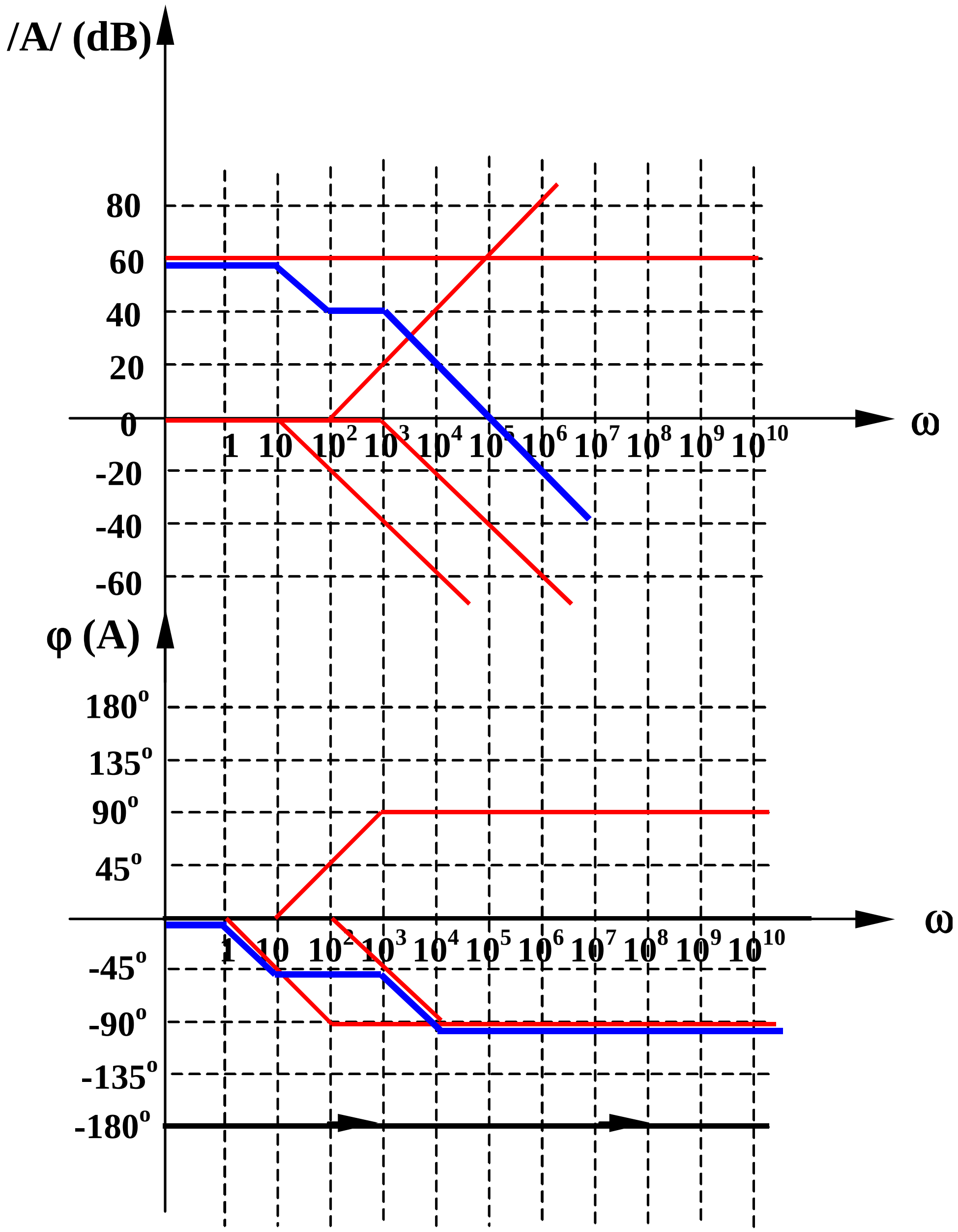




$$T = \frac{V_{tr}}{V_t} = \frac{a\Delta v}{V_t} = a\frac{R_1}{R_1 + R_3 + R_2 // X_{C1}}$$

$$T = a\frac{R_1}{R_1 + R_3 + \frac{R_2}{1 + j\omega C_1 R_2}} = a\frac{R_1}{R_1 + R_2 + R_3}\frac{1 + j\omega C_1 R_2}{1 + j\omega C_1\left[R_2 // \left(R_1 + R_3\right)\right]}$$

$$T = 10^3\frac{1 + j\frac{\omega}{10^2}}{\left(1 + j\frac{\omega}{10}\right)\left(1 + j\frac{\omega}{10^3}\right)}$$

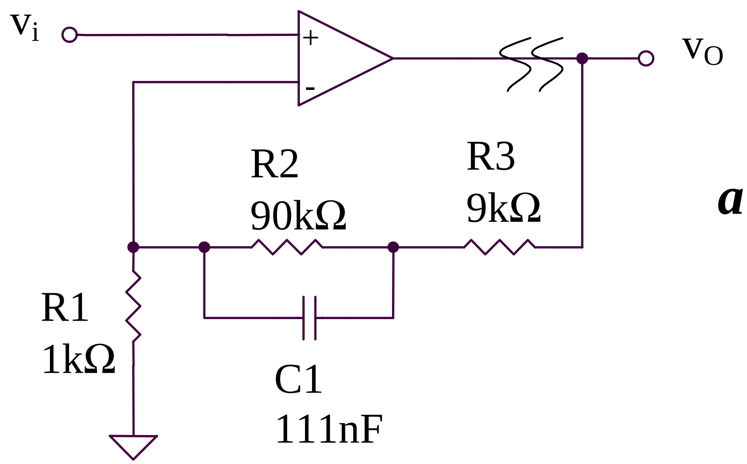


The horizontal line does not intersect the phase diagram, so the circuit is stable.

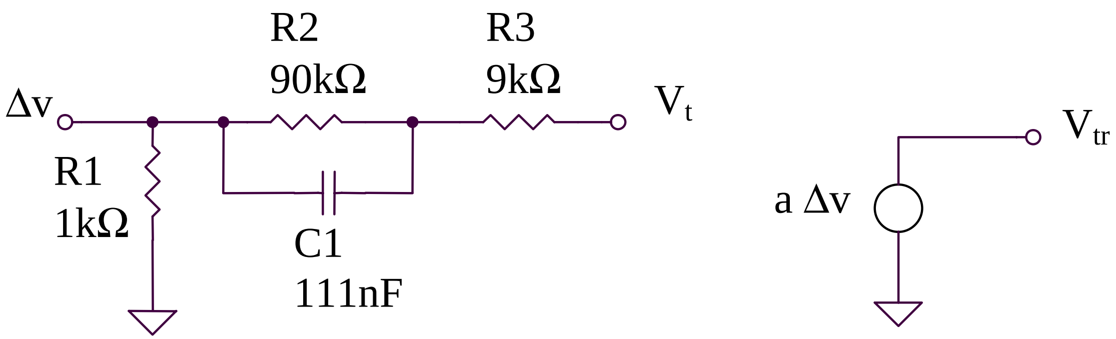
6.4.4. Exemple 3

6.4.4. Exemple 3

Evaluate the stability of the following circuit



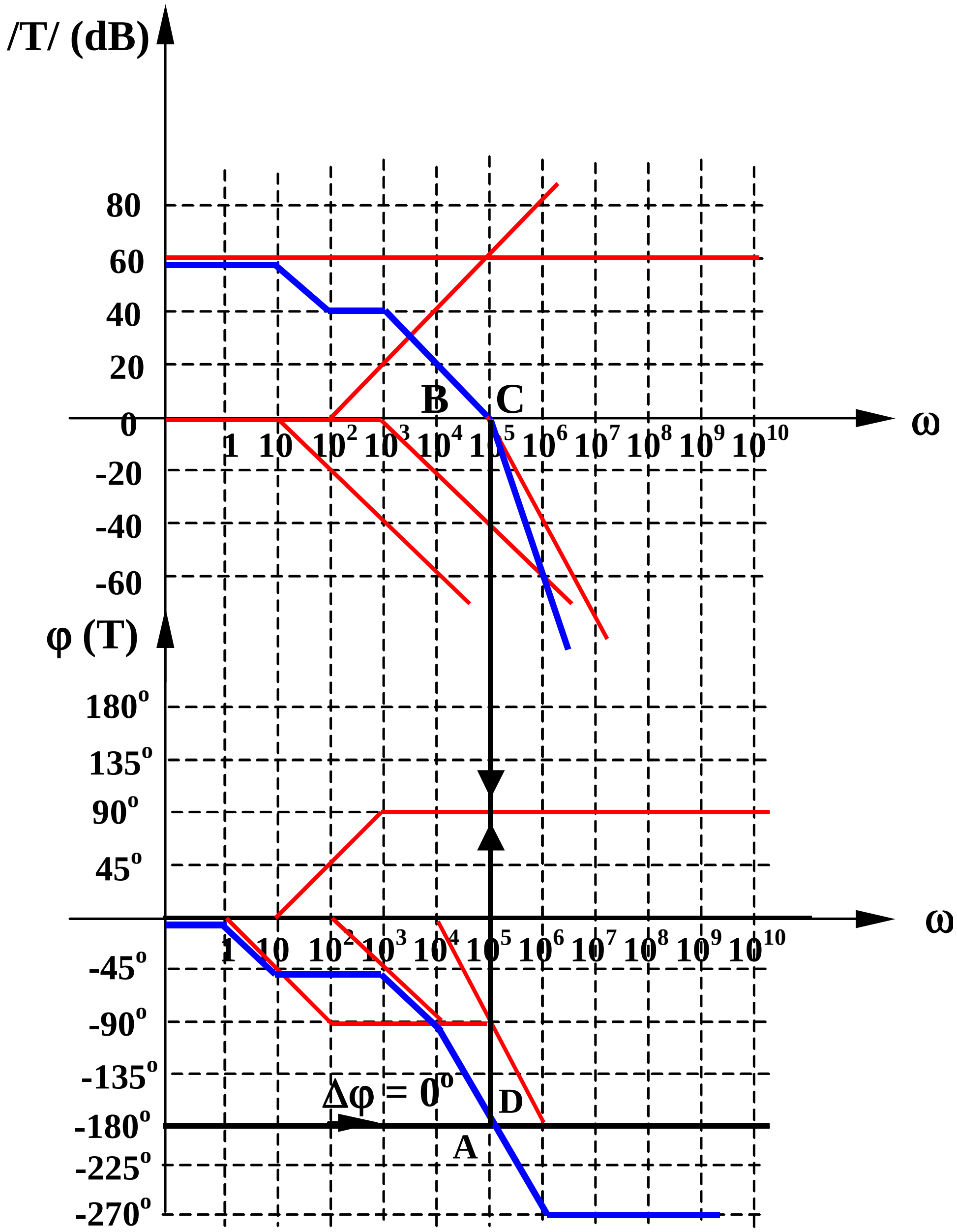
$$a(j\omega) = \frac{10^5}{\left(1 + j\frac{\omega}{10}\right)\left(1 + j\frac{\omega}{10^5}\right)^2}$$



$$T = \frac{V_{tr}}{V_t} = \frac{a\Delta v}{V_t} = a\frac{R_1}{R_1 + R_3 + R_2 \mathbin{//} X_{C1}}$$

$$T = a\frac{R_1}{R_1 + R_3 + \dfrac{R_2}{1 + j\omega C_1 R_2}} = a\frac{R_1}{R_1 + R_2 + R_3}\frac{1 + j\omega C_1 R_2}{1 + j\omega C_1\left[R_2 \mathbin{//} \left(R_1 + R_3\right)\right]}$$

$$T = 10^3 \frac{1 + j\dfrac{\omega}{10^2}}{\left(1 + j\dfrac{\omega}{10}\right)\left(1 + j\dfrac{\omega}{10^3}\right)\left(1 + j\dfrac{\omega}{10^5}\right)^2}$$



The horizontal line at –180^O intersects the phase diagram in A point, $/T_B/ = 0$, so the circuit is at the stability limit (Δφ = 0).