Chapter 1
Models of bipolar and MOS devices

1.1. Fundamental relations for the bipolar transistor

1.1.1. Large signal, normal active region

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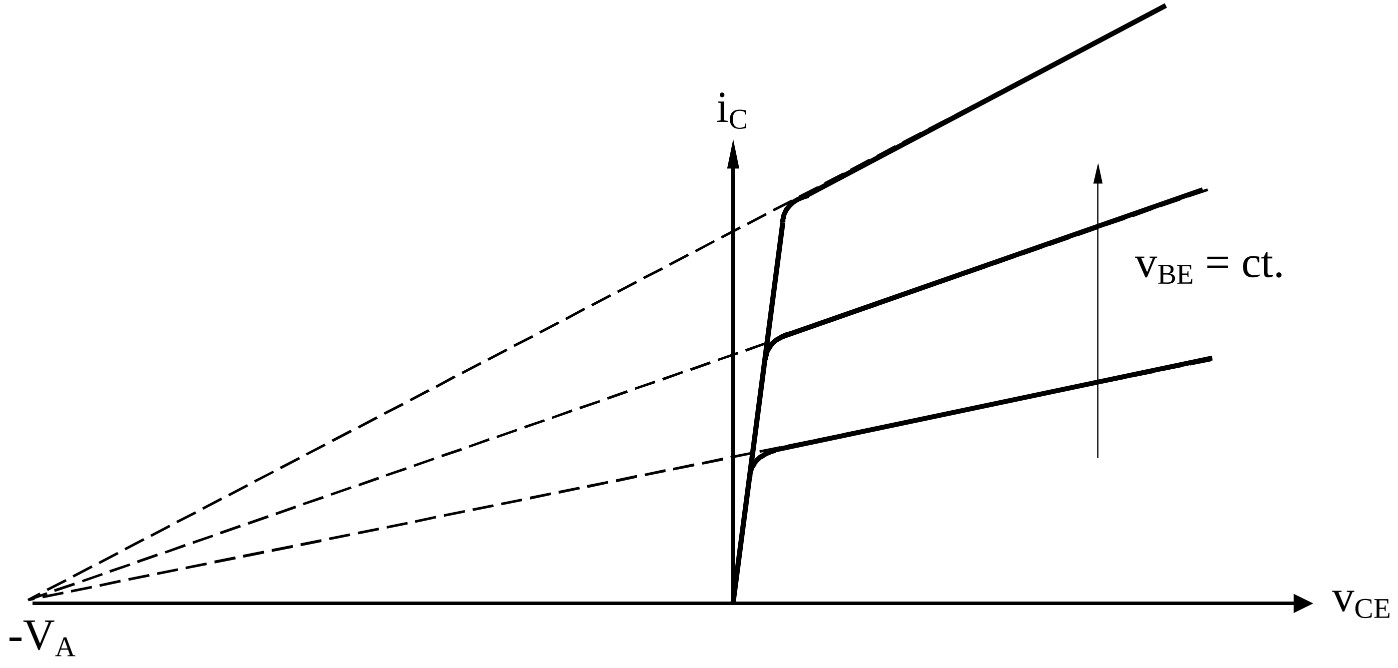
The relation between i_c and v_{BE}:

$$i_c = I_S\left(e^{\frac{v_{BE}}{V_{th}}} - 1\right) \cong I_S e^{\frac{v_{BE}}{V_{th}}}$$

The Early effect:

$$i_c = I_S e^{\frac{v_{BE}}{V_{th}}}\left(1 + \frac{v_{CE}}{V_A}\right)$$



1.1.2. The small signal models of bipolar transistor at normal active region

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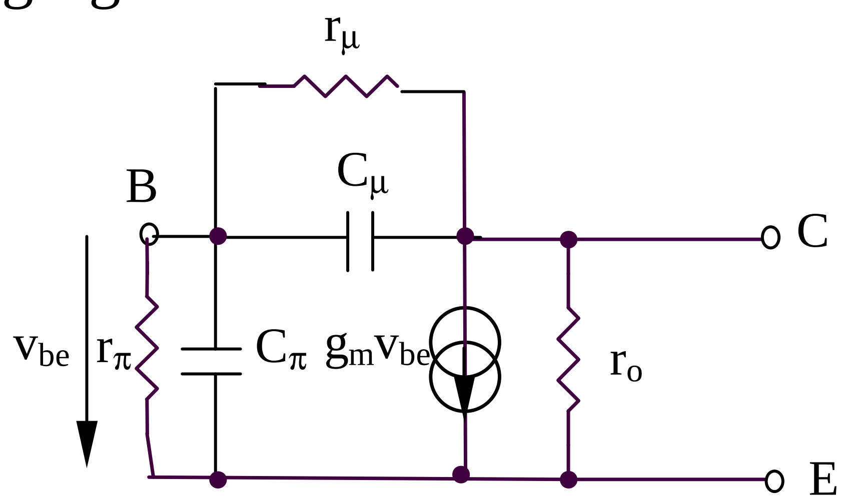
The small signal model of the bipolar transistor is presented in the following figure:



Transfer conductance:

$$\left.\begin{aligned} g_m &= \frac{\partial I_C}{\partial V_{BE}} \\ I_C &\cong I_S \exp\left(\frac{V_{BE}}{V_{th}}\right) \end{aligned}\right\} \Rightarrow g_m = I_S \exp\left(\frac{V_{BE}}{V_{th}}\right)\frac{1}{V_{th}} = \frac{I_C}{V_{th}} \cong 40 I_C$$

Output resistance:

$$\left.\begin{array}{l} r_o = \dfrac{1}{g_o} = \dfrac{1}{\dfrac{\partial I_C}{\partial V_{CE}}} \\ I_C = I_S \exp\left(\dfrac{V_{BE}}{V_{th}}\right)\left(1 + \dfrac{V_{CE}}{V_A}\right) \end{array}\right\} \Rightarrow r_o = \frac{1}{I_S \exp\left(\dfrac{V_{BE}}{V_{th}}\right)\dfrac{1}{V_A}} \cong \frac{V_A}{I_C}$$

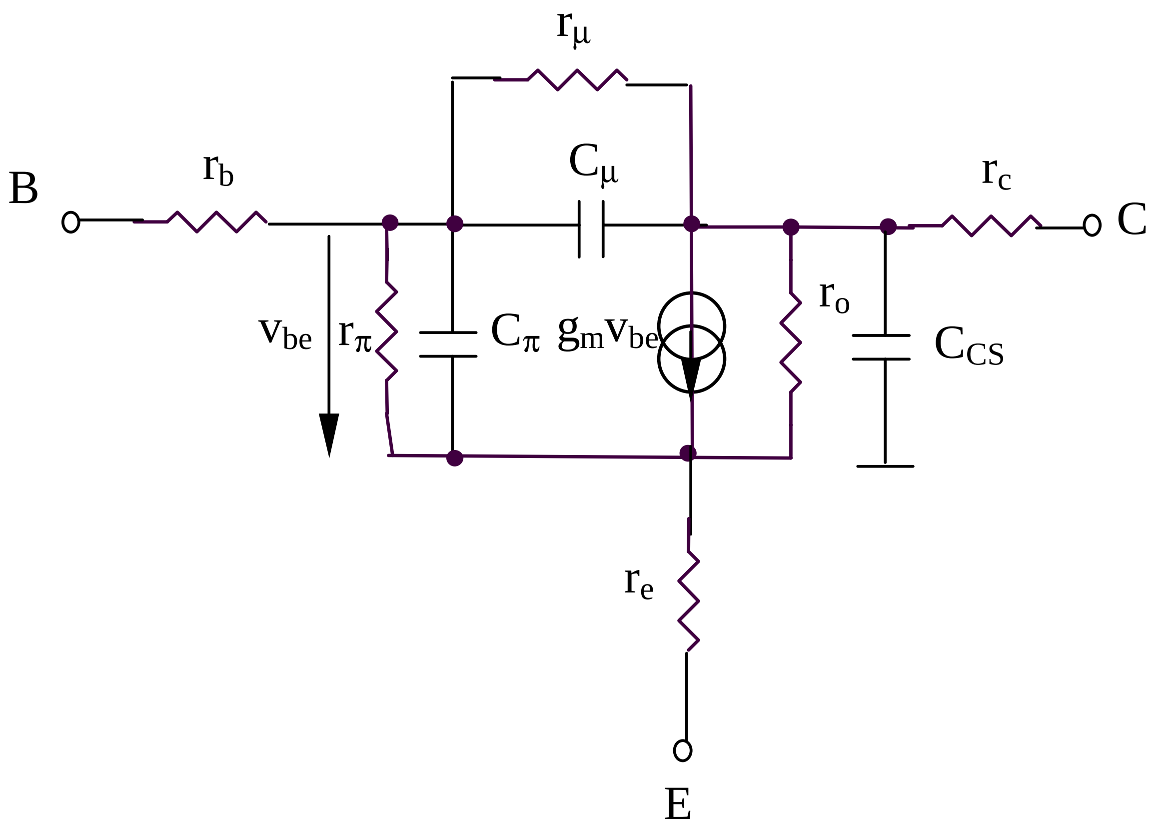
Resistance r_π:

$$r_\pi = \frac{\partial v_{BE}}{\partial i_B} = \frac{\beta}{g_m}$$

Resistance r_μ:

$$r_\mu = \frac{\partial v_{CE}}{\partial i_C} = K\beta_O r_O$$

More complete equivalent circuit:



Example:

$$I_C = 1mA, \beta = 100, V_A = 100V$$

$$\Rightarrow g_m = 40 I_C = 40mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40mA/V} = 2.5k\Omega$$

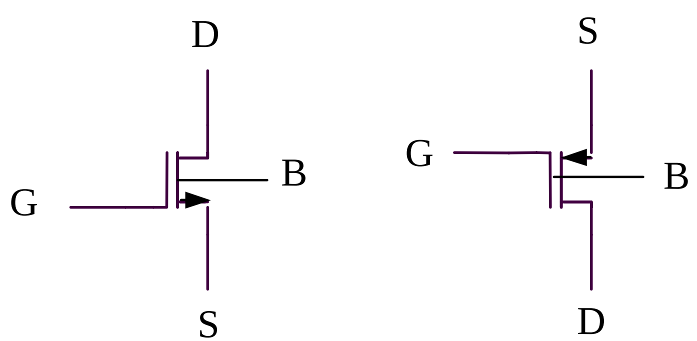
$$r_o = \frac{V_A}{I_C} = \frac{100V}{1mA} = 100k\Omega$$

1.2. Fundamental relations for MOS transistor

1.2.1. Large signal

Symbols:

NMOS **PMOS**



Notations:
G = gate
D = drain
S = source
B = bulk

W = channel width
L = channel length
(W/L = aspect ratio)
K' = transconductance parameter
V_T = threshold voltage
V_{GS} = gate-source voltage
V_{DS} = drain-source voltage

1.2.1. Large signal

I. Strong inversion region

$$V_{GS} > V_T$$

a. Saturation

$$V_{DS} \geq V_{DSsat} = V_{GS} - V_T$$

$$I_D = \frac{K}{2}\left(V_{GS} - V_T\right)^2 \qquad K = K'\frac{W}{L} = \mu_n C_{ox}\frac{W}{L}$$

b. Linear region

$$V_{DS} < V_{DSsat}$$

$$I_D = K\left[\left(V_{GS} - V_T\right) - \frac{V_{DS}}{2}\right]V_{DS}$$

II. Weak inversion region

$$V_{GS} < V_T$$

$$I_D = I_{D0}\frac{W}{L}\exp\left(\frac{V_{GS} - V_T}{nV_{th}}\right)$$

$$I_D\big|_{sat} = I_D\big|_{w.i.}$$

$$\left.\frac{\partial I_D}{\partial V_{GS}}\right|_{sat} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{w.i.}$$

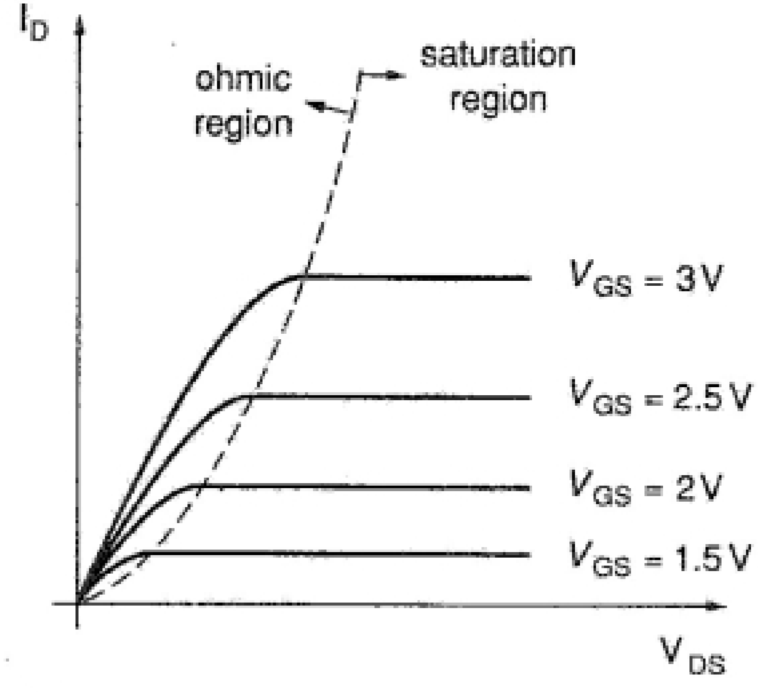
$$\frac{K}{2}(V_{GS} - V_T)^2 = I_{D0}\frac{W}{L}\exp\left(\frac{V_{GS} - V_T}{nV_{th}}\right)$$

$$K(V_{GS} - V_T) = I_{D0}\frac{W}{L}\frac{1}{nV_{th}}\exp\left(\frac{V_{GS} - V_T}{nV_{th}}\right)$$

$$\Rightarrow \begin{cases} \dfrac{V_{GS} - V_T}{2} = nV_{th} \\ \dfrac{K}{2}(2nV_{th})^2 = I_{D0}\dfrac{W}{L}e^2 \end{cases}$$

$$\Rightarrow \begin{cases} V_{GS} = V_T + nV_{th} \\ I_{D0} = \dfrac{K'2(nV_{th})^2}{e^2} \end{cases}$$

Output characteristics of MOS transistor



Second-order effects:

a. Channel-length modulation

$$I_D = \frac{K}{2}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

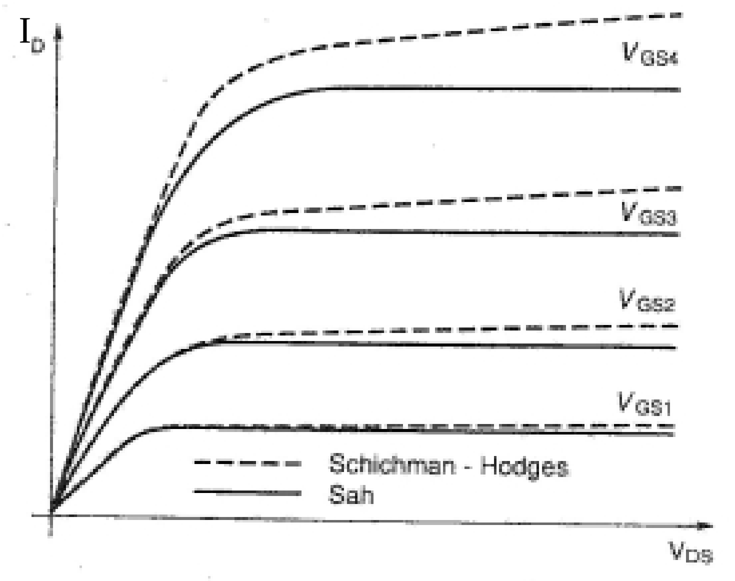
b. Mobility degradation

$$K = \frac{K_0}{[1 + \theta_G(V_{GS} - V_T)](1 + \theta_D V_{DS})}$$

c. Bulk effect

$$V_T = V_{T0} + \gamma\left(\sqrt{\Phi - V_{BS}} - \sqrt{\Phi}\right)$$

Channel-length modulation



1.2.2. Small signal model of MOS transistor

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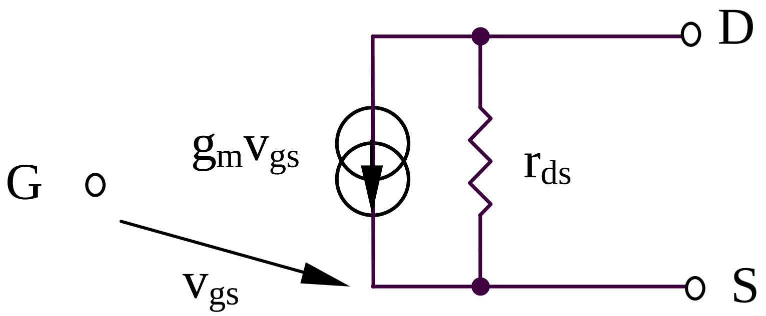


$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

$$r_{ds} = \frac{1}{g_{ds}} = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}}$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} \cong K(V_{GS} - V_T)$$

$$I_D = \frac{K}{2}(V_{GS} - V_T)^2 \Rightarrow V_{GS} - V_T = \sqrt{\frac{2I_D}{K}}$$

$$\Rightarrow g_m = \sqrt{2KI_D}$$

$$r_{ds} = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\frac{K}{2}(V_{GS} - V_T)^2 \lambda} \cong \frac{1}{\lambda I_D}$$

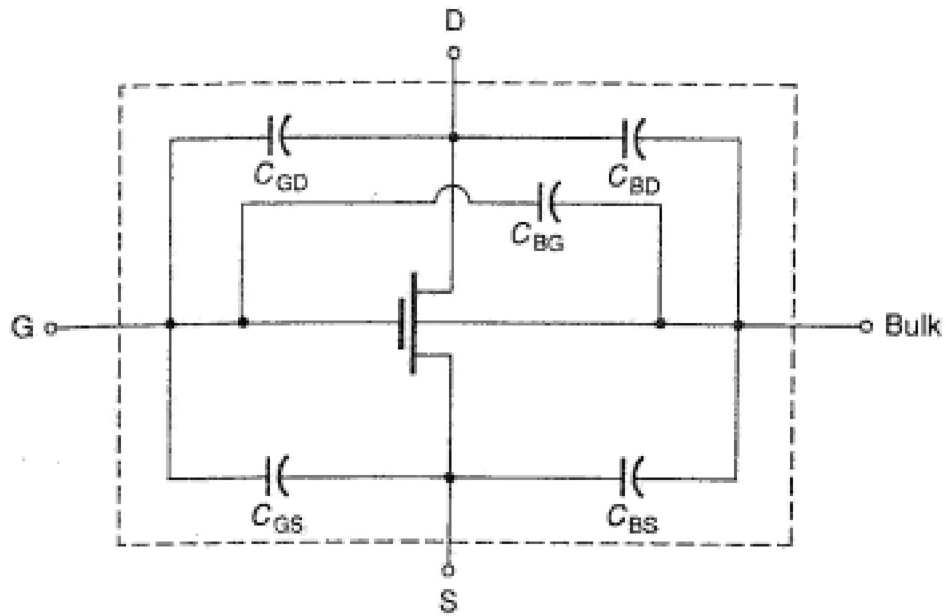
Example

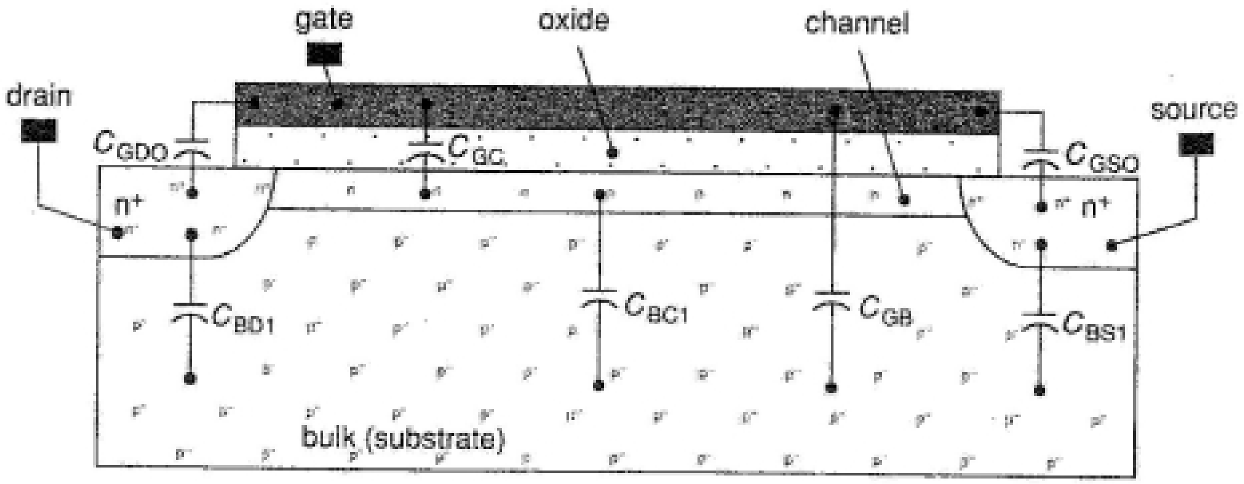
$$I_D = 1mA, \lambda = 10^{-3}V^{-1}, K = 5 \times 10^{-4} A/V^2$$

$$\Rightarrow g_m = \sqrt{2KI_D} = 1mA/V$$

$$r_{ds} = \frac{1}{\lambda I_D} = 1M\Omega$$

High frequency model





1.3. Dynamic resistances

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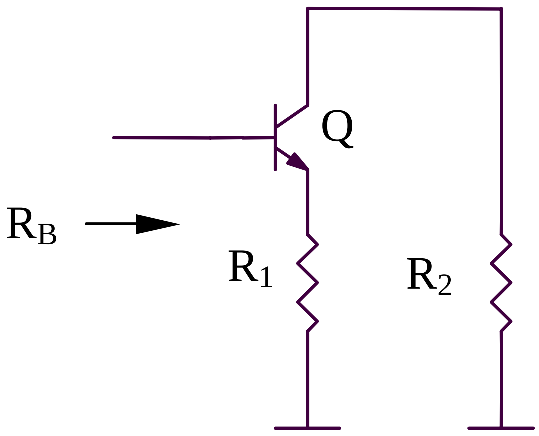
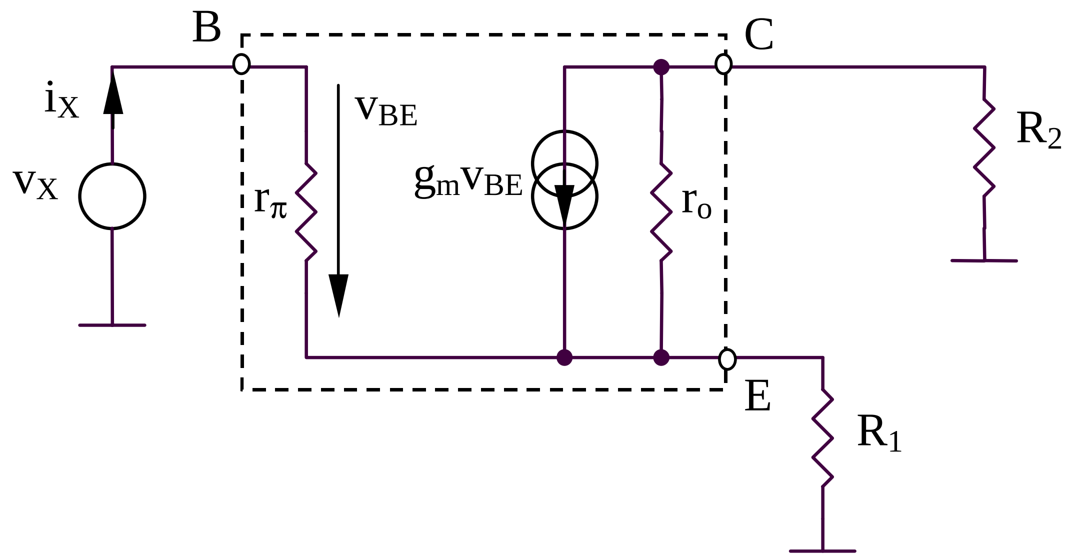
Resistance view in base

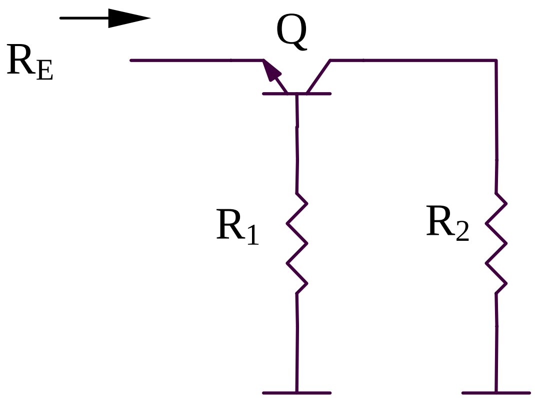


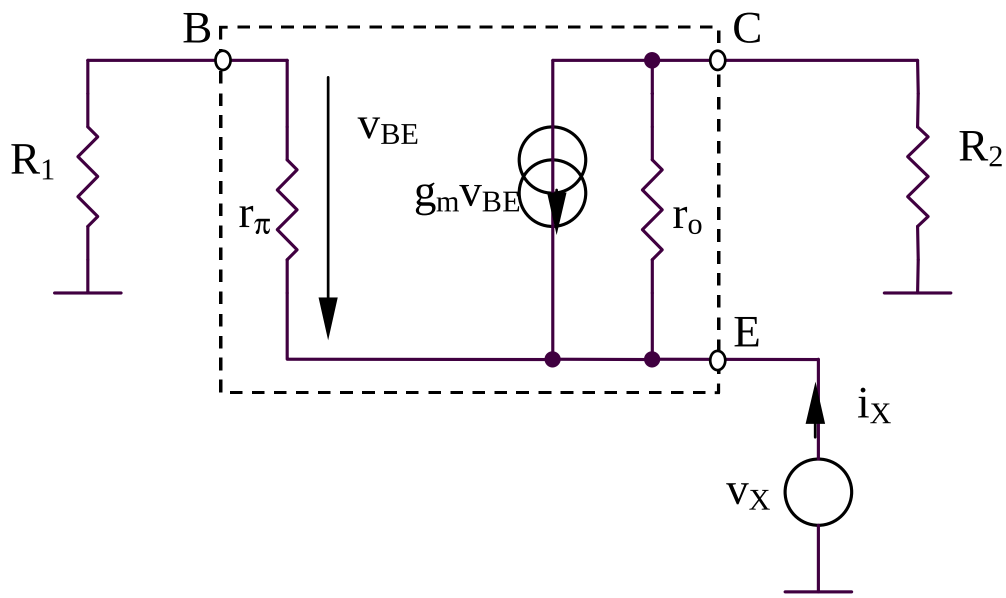


$$v_x = i_x r_\pi + (\beta + 1) i_x R_1$$

$$R_B = \frac{v_x}{i_x} = r_\pi + (\beta + 1) R_1$$

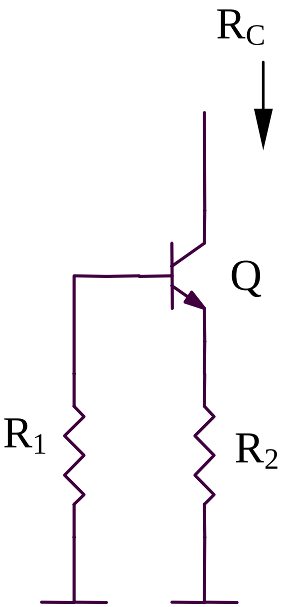
Resistance view in emitter

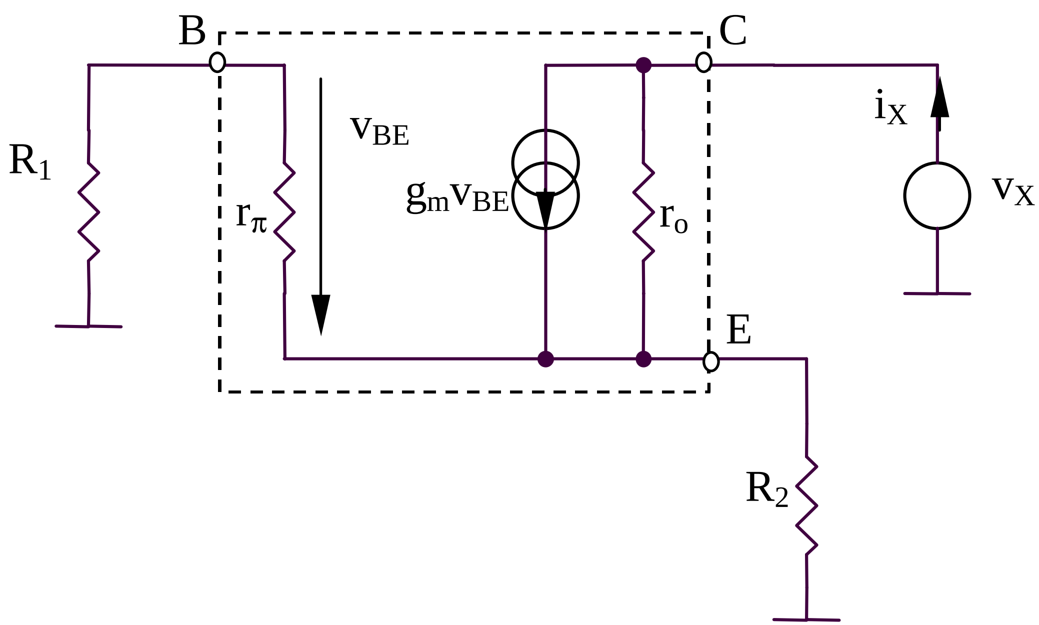




$$R_E = \frac{R_1 + r_\pi}{\beta + 1}$$

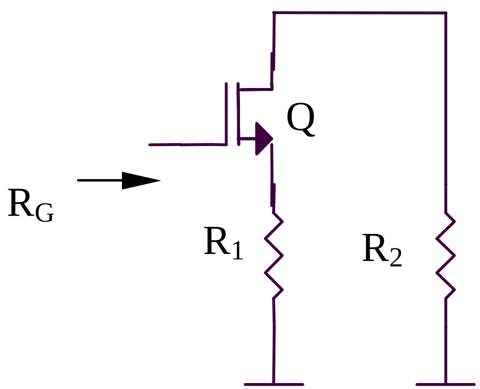
Resistance view in collector

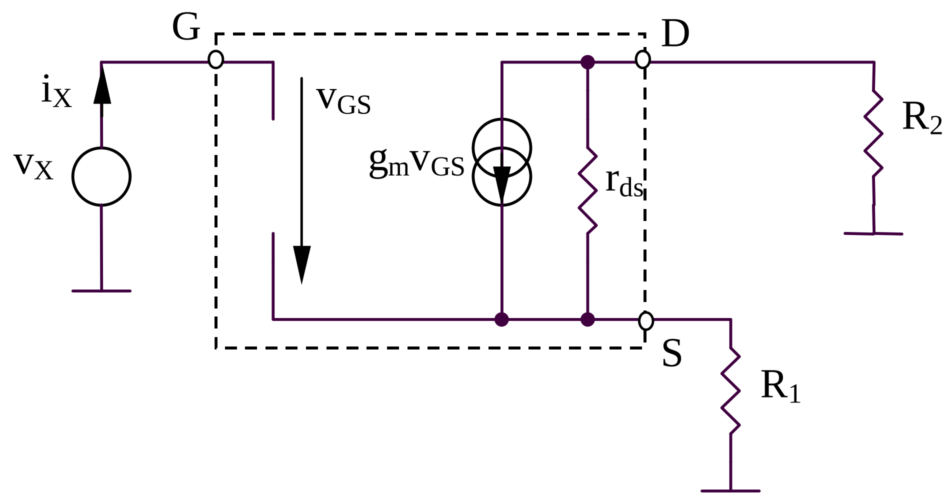




$$R_C = r_o\left(1 + \frac{\beta R_2}{r_\pi + R_1 + R_2}\right)$$

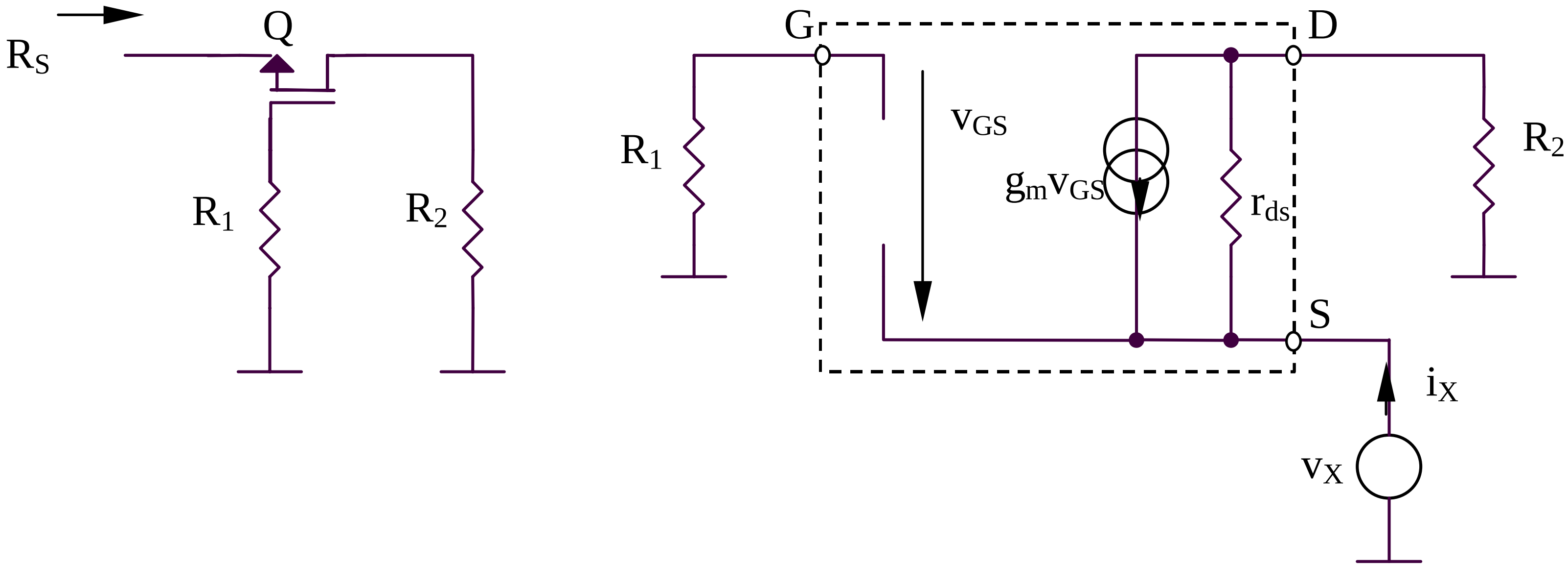
Resistance view in the gate





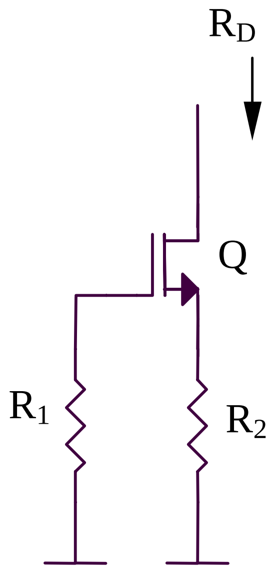
$$R_G = \infty$$

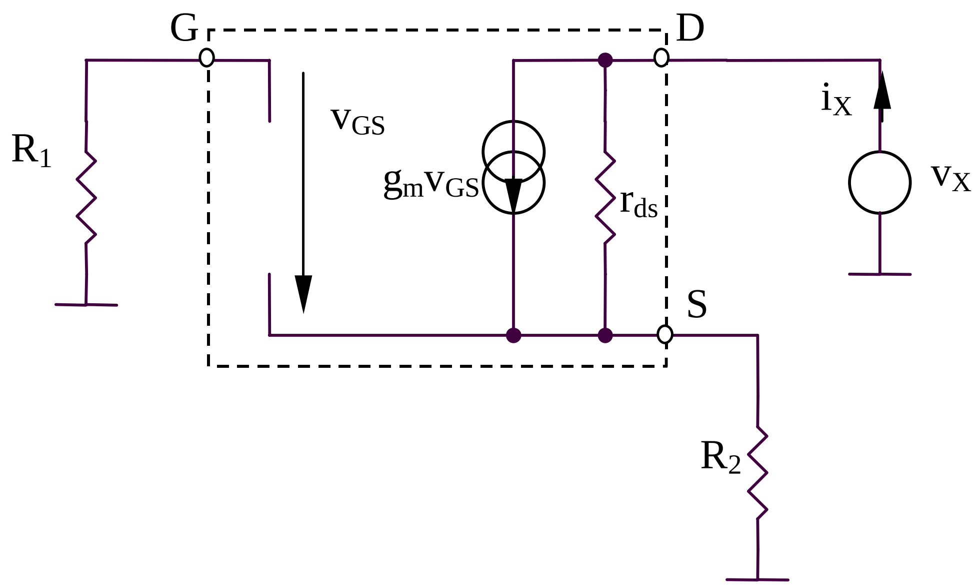
Resistance view in source



$$i_x \cong -g_m v_{gs} = g_m v_x \Rightarrow R_S = \frac{v_x}{i_x} = \frac{1}{g_m}$$

Resistance view in drain





$$\left.\begin{aligned} v_x &= \left(i_x - g_m v_{gs}\right) r_{ds} + i_x R_2 \\ v_{gs} &= -i_x R_2 \end{aligned}\right\} \Rightarrow R_D = \frac{v_x}{i_x} = r_{ds}\left(1 + g_m R_2\right) + R_2 \cong r_{ds}\left(1 + g_m R_2\right)$$